Chapter Five

Non-Concurrent, Non-Parallel, Coplanar Forces

Equilibrium Conditions

Non-concurrent Forces: (i) The algebraic sum of all the horizontal components of the forces must be equal to zero, i.e. the total sum to the *left* must equal the sum to the *right*.
(ii) The algebraic sum of all the vertical components of the forces must be equal to zero, i.e. the total sum *upwards* must equal the total sum *downwards*.

(iii) The two conditions given for concurrent forces must be satisfied and, in addition, the algebraic sum of all the moments taken about any point in the plane of the forces must equal zero, i.e. the sum of all the *clockwise* moments must equal the sum of all the *anticlockwise* moments.

Principles of Equilibrium: Though there are many principles of equilibrium, yet the following three are important from the subject point of view:

Two force principle: As per this principle, if a body in equilibrium is acted upon by two forces, then they must be equal, opposite and collinear.

Three force principle: As per this principle, if a body is in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with the third force.

Four force principle: As per this principle, if a body is in equilibrium is acted upon by four forces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two forces.

Two-force Member & Three-force Member: Two general geometric equilibrium conditions serving to indicate rigid bodies (which may be regarded as particles for the sake of their static analysis) may be stated –

(1) Any rigid body under the influence of force actions introduced at only two physical locations will undergo a motion until these (net) force actions comprise a collinear force system.*

(2) Any rigid body under the influence of force actions introduced at three physical locations will undergo a motion until these (net) force actions comprise a concurrent force system.**

[*a structural member is a slender two force member if – (i) it has uniform cross-section, and is longer than it is wide, (ii) it is connected only to pin joints, or ball and socket joints, at its ends (iii) it has negligible weight compared with the forces acting at its ends]

[**the only exception in the case of three force action lines becoming parallel. Many authors regard this specialized non-concurrent case as an illustration of a third class of force system – the parallel force system]

These conditions are commonly phrased in such a way that the student thinks they have to do with the number of forces, rather than the number of places of force application. For example, the first condition identifies rigid bodies commonly classified as two-force members; the second identifies rigid bodies similarly classified as three-force members.

Characteristically, the two-force members are encountered in simple trusses, guyed bodies, and bodies having braces or struts. Similarly, the three-force members are encountered in dealing with simple machines, frames, and some classes of fasteners.

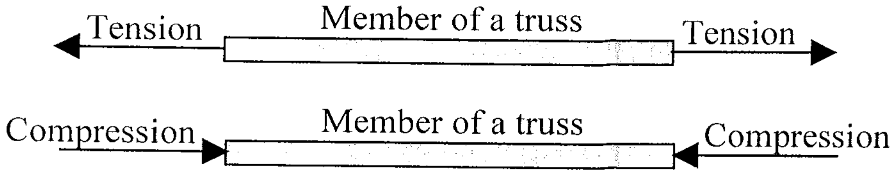
Truss: A truss is a framed or jointed structure designed to act as a beam while each member is usually subjected to longitudinal stress only, either tension or compression. An Italian architect, Andrea Palladio (1518-1580), is believed to have developed the first trusses.

A truss is defined as a structure formed by a group of members arranged in the shape of one or more triangles. Because the members are assumed to be connected with frictionless pin, the triangle is the only stable shape.

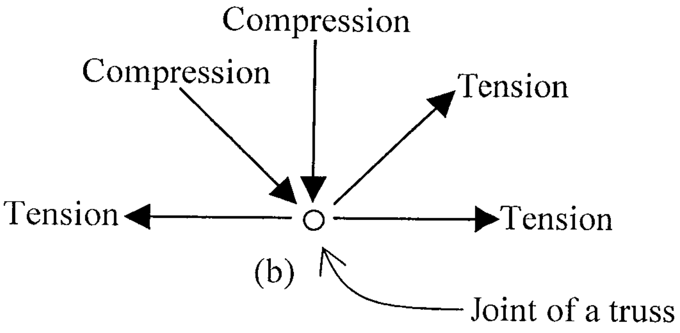
Plane truss: All the truss members lie in same plane and load on the truss also lie in the same plane thus forming a 2- D (two dimensional) planar structure.
Space truss: All the truss members and load on the truss are not lying in the same plane thus forming a 3- D (three dimensional) space structure.

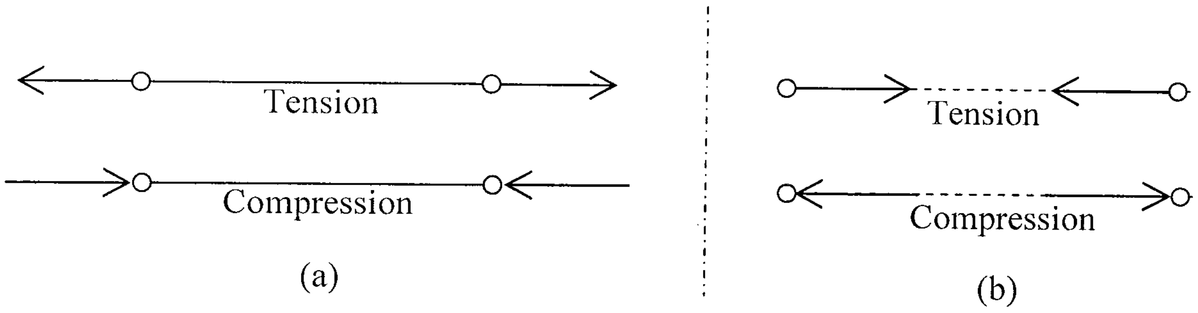
Conventions and Notations: The unknown member forces (reactions) are assumed to be tensile (positive); the forces are thus directed away from the connecting joints. The member force is compressive, in case it is found to be negative. The force in a member is denoted by *F* with two subscripts indicating the connecting joints; F_{ij} indicates the force in a member connected to joints i and j. The subscripts can be written in any order; F_{ij} and F_{ji} are equivalent. Further, the subscripts may be Roman letters or Arabic numerals.

Representation of Bar Forces: There are two ways in which it is convenient to represent a bar force. In the first, the force that is exerted on the bar by the joint is indicated while in the second, the action of the bar on the joint is shown. These are shown in Figure 1.



(a)





(a)

(b)

Figure 1: Bar force representation
(a) action of joints on bar
(b) action of bar on joints

In the first method, a free-body diagram of the bat is considered and the forces that the joints apply to the bar ends are shown. In the second approach, a free-body diagram of the joint is considered and the action of the bar on the joint is shown.

The two bar force representations are equivalent – each has advantages in certain situations and both will be used.

Assumptions Made in Truss Analysis: The term truss analysis is used to describe the process whereby the bar forces in a truss are determined. The following assumptions are made in order to simplify the analysis of trusses:
Assumption 1: Truss members are connected together with frictionless pin. (Pin connections are used for few trusses erected today, and no pins and frictionless. A heavy bolted or welded joint is a far cry from a frictionless pin.)
Assumption 2: Truss members are straight linear elements of relatively small cross section compared to the truss dimension. (If they were not straight, the axial forces would cause them to have bending moments.)

Assumption 3: The deformation of the truss under load, caused by the change in lengths of the individual members, are not of sufficient magnitude to cause appreciable changes in the overall shape and dimensions of the truss. Special consideration may have to be given to some very long and flexible trusses.
Assumption 4: Members are so arranged that the loads and reactions are applied only at the truss joints.
Assumption 5: The dead load of the truss member which is distributed along their lengths. These loads are generally small and are lumped at the joints for purpose of analysis.
Assumption 6: The centroidal axis of each member is straight and coincides with the line connecting the joint centers at each end of the member.

Method of analysis of plane trusses:

(1) Method of joints
(2) Method of sections

Method of Joints:

1. Determine the support reactions from equilibrium conditions as the in the case of beams.
2. Select a joint with only two members. Formulate two equations of equilibrium so that only one unknown appears in each equation, if possible.
3. Select that the next joint such that only two unknown member forces exist, and solve them as in the previous step.
4. Repeat the previous steps till all the member forces are obtained.

Method of Sections or Method of Shear & Moment:

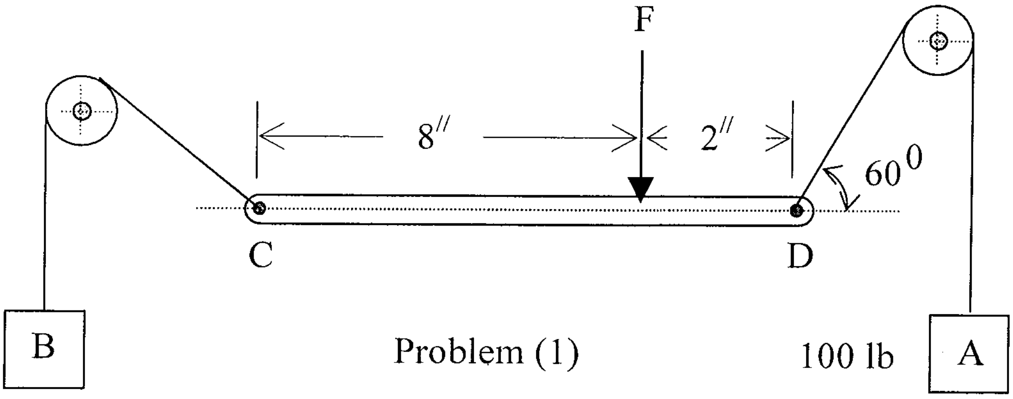
1. Determine the support reactions from equilibrium conditions as the in the case of beams.
2. Pass a section through the truss depth to intersect the required members; note the points of intersection of member forces, if any.
3. Take moments of all the forces on the free body diagram of the segment isolated by the section about the point of concurrence of member forces, if any. The solution of the moment equation yields the non-concurrent member force. Force equilibrium conditions can be applied, where required.
4. Repeat steps 2 and 3 for other member forces.

In many cases, it is found that a combination of the method of joints and the method of sections is employed. One should determine the desired bar forces by employing the laws of statics on the most convenient free-body diagram, and should not be committed to some particular method.

Member of zero forces: It is possible that some of the truss members do not carry any forces for a given system of loading. Identifying such members would simplify the analysis considerably. The basic principles of statics provide guidelines in identifying the members with zero forces for a given system of forces.

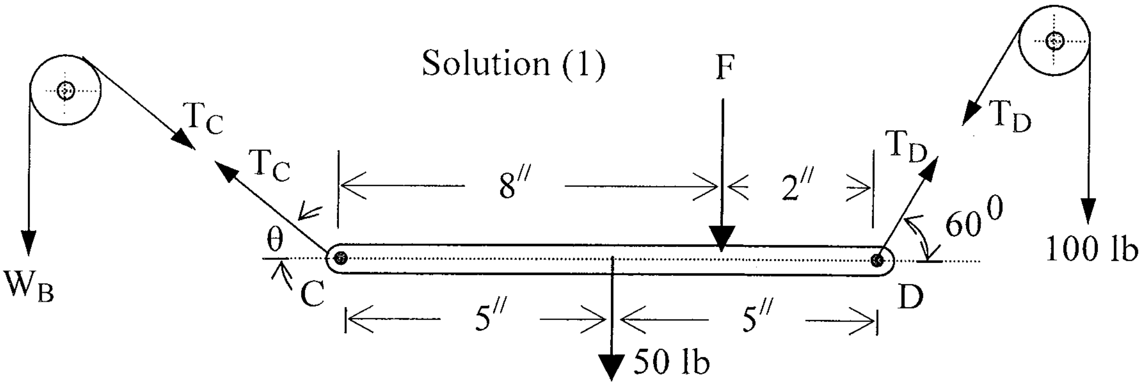
Problems

Problem (1): Weight A of 100 lb, CD is a rigid, uniform body weighing 50 lb, and the pegs are smooth. If CD remains in a horizontal position, determine F and the weight B for equilibrium.
Answer (1): F = 77.00317547 lb (↓); weight B = 64.2822784 lb (↓)



Problem (1)

Solution (1): No friction at the pegs; W_B (↓) = T_C & $T_D = W_A$ (↓) = 100 lb.



Solution (1)

$\sum M_C = 0$ (clockwise +ve); $+ 50\times5 + F\times8 - T_D\times \sin 60^0\times10 = 0$

$\Rightarrow + 50\times5 + F\times8 - 100\times \sin 60^0\times10 = 0$

$\Rightarrow F = + 77.00317547$ lb. (↓)

$\sum F_X = 0$ (→+ve); $+ T_D\times \cos 60^0 - T_C\times \cos \theta = 0$

$\Rightarrow T_C \cos \theta = T_D\times \cos 60^0 = 100\times \cos 60^0 = 50$ lb.

$\sum F_Y = 0$ (↑+ve); $T_D\times \sin 60^0 - F - 50 + T_C\times \sin \theta = 0$

$\Rightarrow T_C \sin \theta = F + 50 - T_D\times \sin 60^0 = + 77.00317547 + 50 - 100\times \sin 60^0$

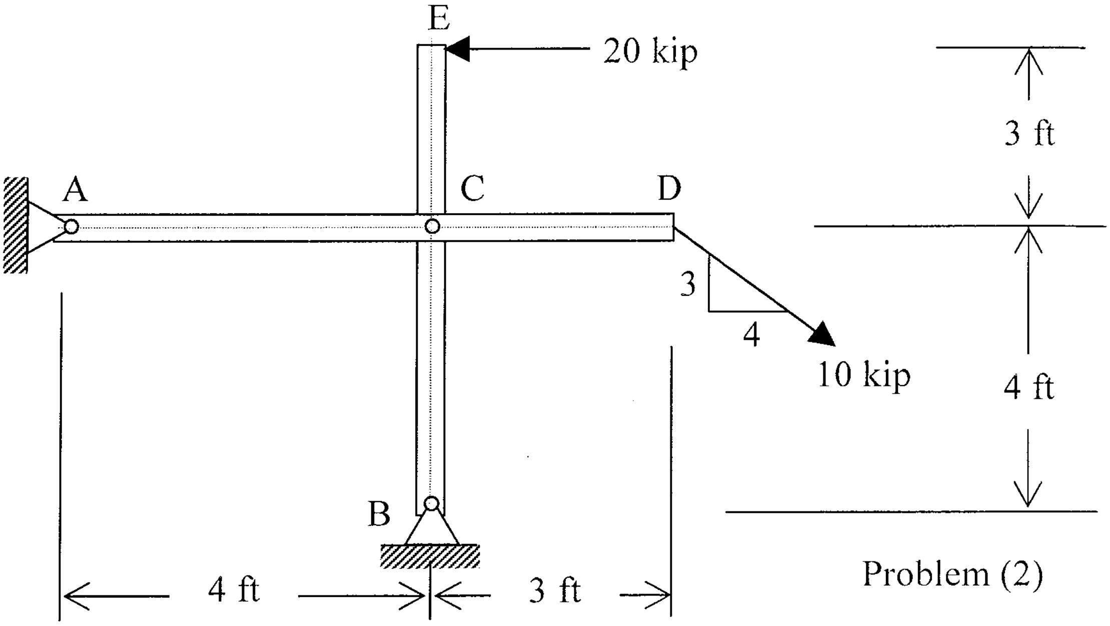
$\Rightarrow T_C \sin \theta = 40.40063509$ lb.

$$T_C = \sqrt{(T_C \cos\theta)^2 + (T_C \sin\theta)^2} = \sqrt{(50)^2 + (40.40063509)^2}$$

$\Rightarrow T_C = 64.2822784$ lb. (tension); $\Rightarrow W_B\,(\downarrow) = T_C = 64.2822784$ lb.

Problem (2): Determine the support reactions at A & B and internal pin reactions at C.

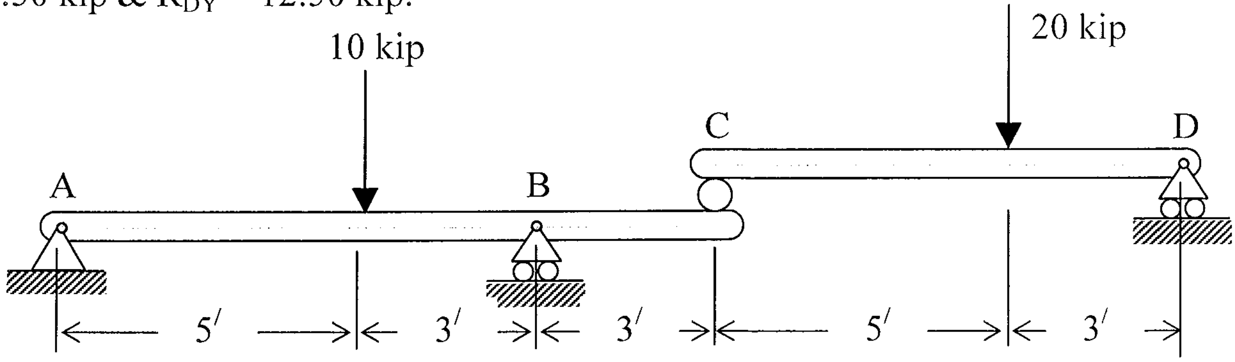
Answer (2): $R_{Ax} = 27$ kip (→), $R_{Ay} = 4.50$ kip (↓), $R_{Bx} = 15$ kip (←), $R_{By} = 10.5$ kip (↑), $R_{Cx} = 35$ kip, $R_{Cy} = 10.5$ kip.



Problem (2)

Problem (3): determine the reactions at A, B, C & D.

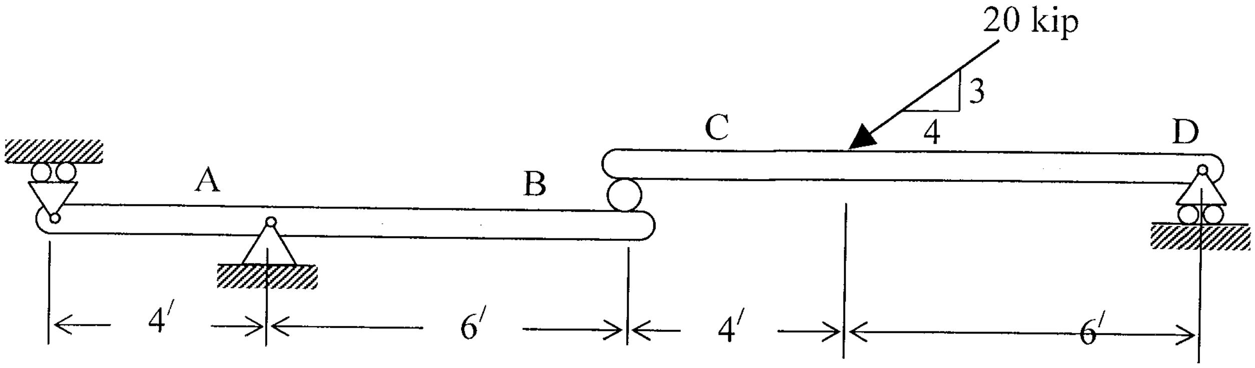
Answer (3): $R_{AX} = 0$ kip, $R_{AY} = 0.9375$ kip (↑), $R_{BY} = 16.5625$ kip (↑), $R_{CY} = 7.50$ kip & $R_{DY} = 12.50$ kip.



Problem (3)

Problem (4): determine the reactions at A, B, C & D.

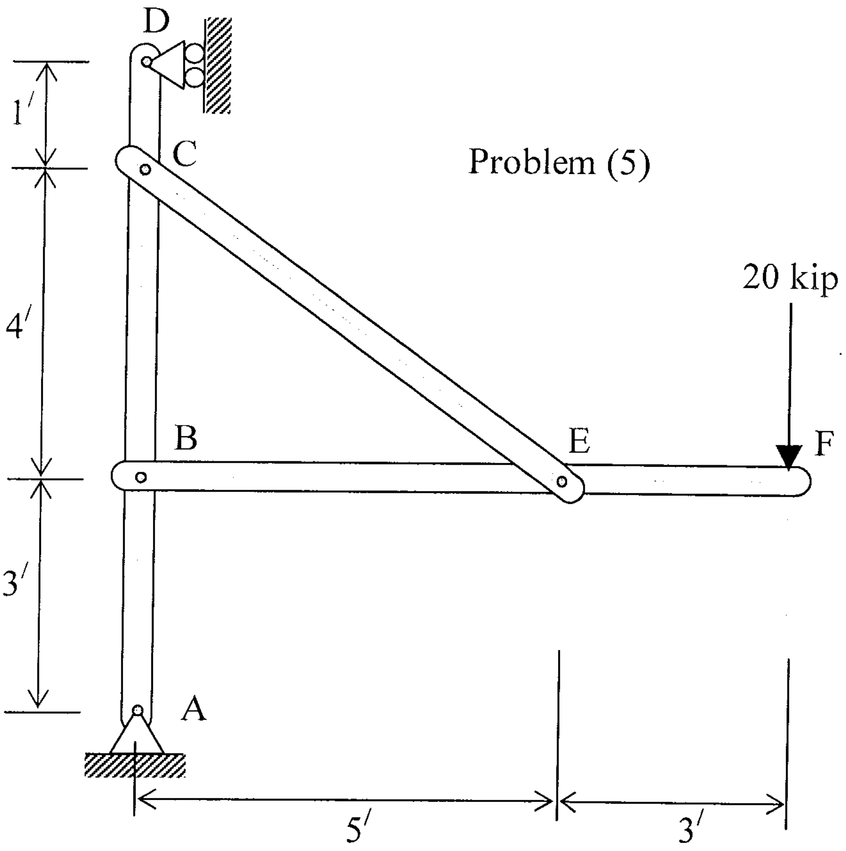
Answer (4): $R_{AY} = 10.80$ kip (↓), $R_{BX} = 0$ kip, $R_{BY} = 18$ kip (↑), $R_{CY} = 7.20$ kip, $R_{DX} = 16$ kip (→) & $R_{DY} = 4.8$ kip (↑).



Problem (4)

Problem (5): Find external support reactions at A & D and internal support reaction at B. Also determine the force in member CE.

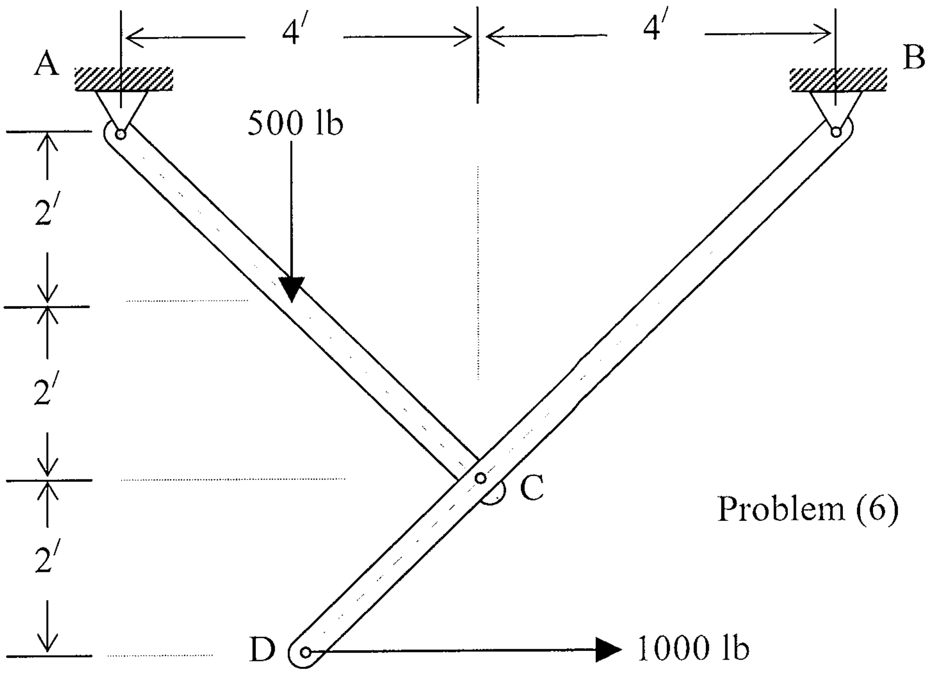
Answer (5): R_{AX} = 20 kip (→), R_{AY} = 20 kip (↑), R_{DX} = 20 kip (←), R_{BX} = 40 kip, R_{BY} = 31.2249 kip & F_{CE} = 51.2249 kip (tension).



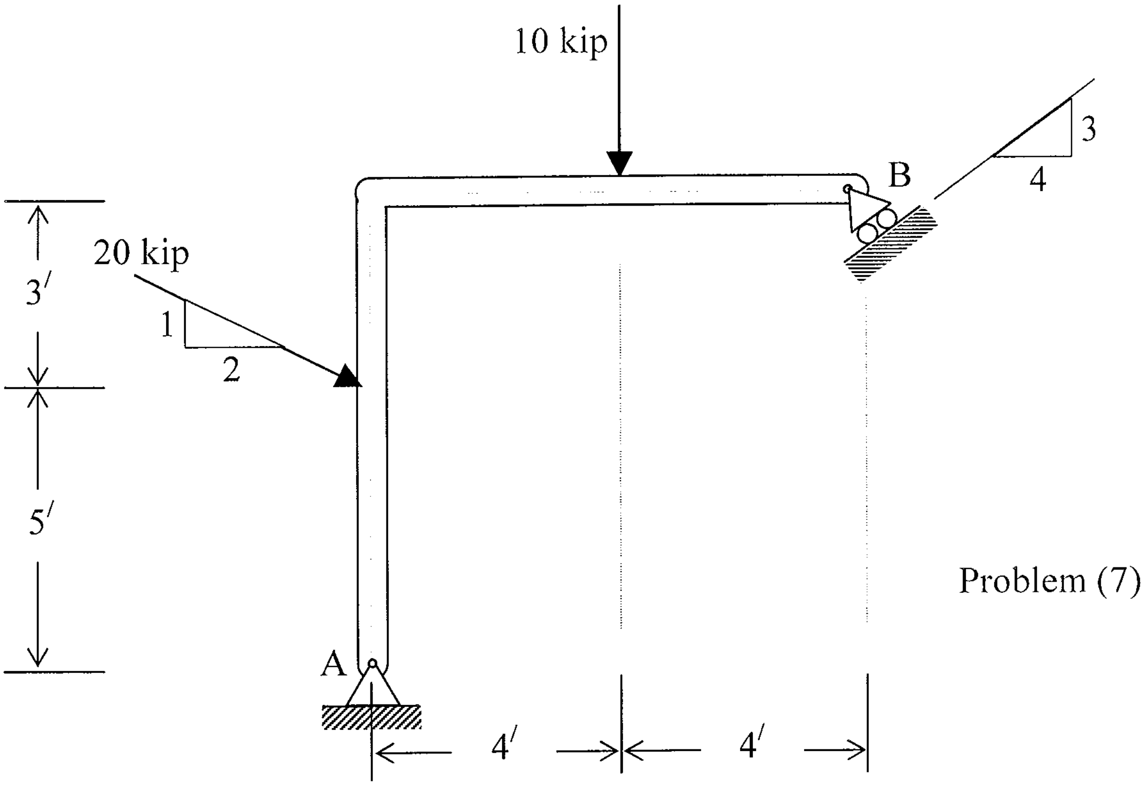
Problem (5)

Problem (6): Find external support reactions at A & B and internal support reaction at C.

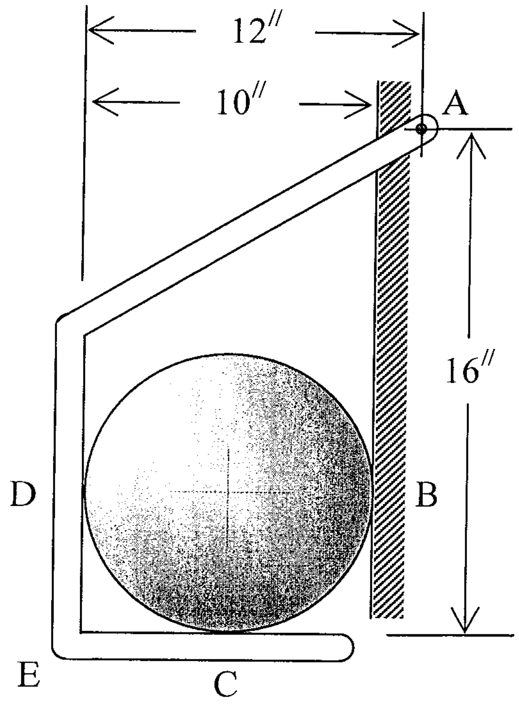
Answer (6): R_{AX} = 875 lb (←), R_{AY} = 1125 lb (↑), R_{BX} = 125 lb (←), R_{BY} = 625 lb (↓), R_{CX} = 875 lb & R_{CY} = 625 lb.



Problem (7): Compute support reactions A and B.
Answer (7): $R_{AX} = 2.0098$ kip (←), $R_{AY} = 9.69638$ kip (↑), $R_B = 11.55738$ kip

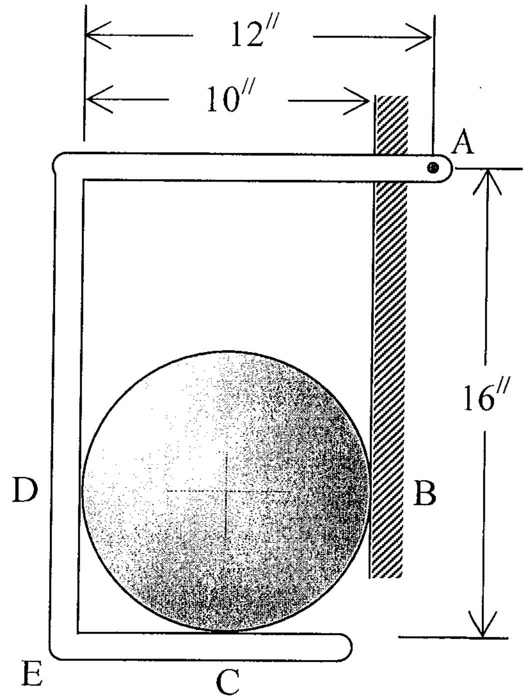


Problem (8): A pipe is supported by hanger rods AE. The hanger is pivoted at pin A. If the pipe section is weighs 800 lb., compute the reactions at A, B, C and D, neglecting the weight of the hanger.
Answer (8): $R_{AX} = 933.3333$ lb (→), $R_{AY} = 800$ lb (↑), $R_B = 933.3333$ lb (←), $R_C = 800$ lb, $R_D = 933.3333$ lb.



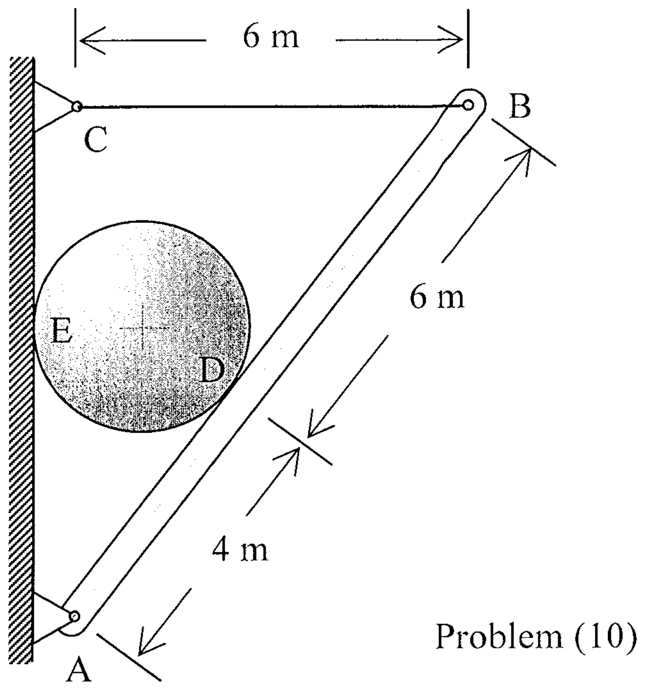
Problem (8)

Problem (9): Same as problem (8) except the weight of pipe section is 1000 lb.
Answer (9): R_{AX} = 1166.6667 lb (→), R_{AY} = 1000 lb (↑), R_B = 1166.6667 lb (←), R_C = 1000 lb, R_D = 1166.6667 lb.



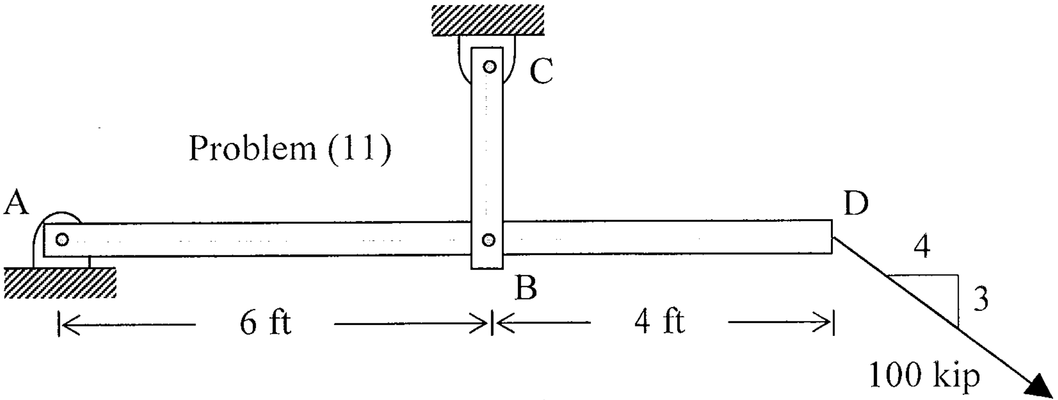
Problem (9)

Problem (10): Determine support reactions at A, E, D and tension in the cable BC. Weight of the cylinder = 500 kN. Neglect the weight of the bar AB.
Answer (10): R_{AX} = 200 kN (←), R_{AY} = 400 kN (↑), R_E = 666.6666 kN (→), R_D = 833.3333 kN, Tension in cable BC = 333.3333 kN.



Problem (10)

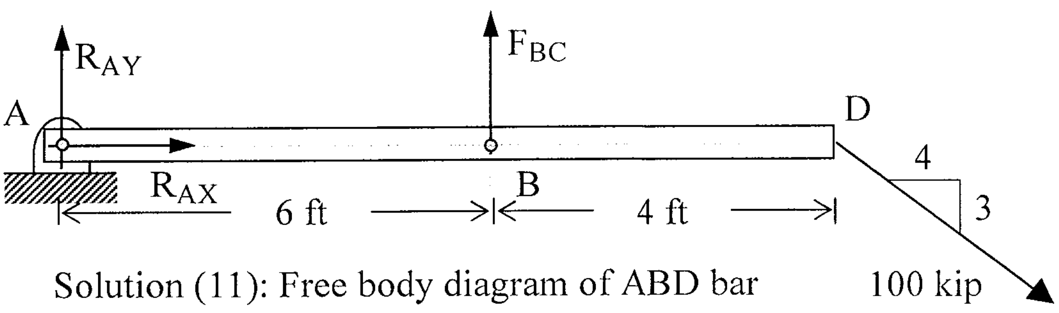
Problem (11): Determine the force in the link BC and support reaction at A.
Answer (11): F_{BC} = 100 kip (tension), R_{AX} = 80 kip (←), R_{AY} = 40 kip (↓).



Problem (11)

Solution (11): Link BC is a two-force member.
From the free-body diagram of bar ABD

$$\sum F_X = 0\ (\rightarrow +ve);\ + R_{AX} + 100 \times \frac{4}{\sqrt{3^2+4^2}} = 0;\ \Rightarrow\ R_{AX} = -\,80 \text{ kip } (\leftarrow)$$



Solution (11): Free body diagram of ABD bar

$$\sum M_A = 0 \text{ (clockwise +ve)}; -F_{BC}\times 6 + 100\times\frac{3}{\sqrt{3^2+4^2}}\times 10 = 0$$

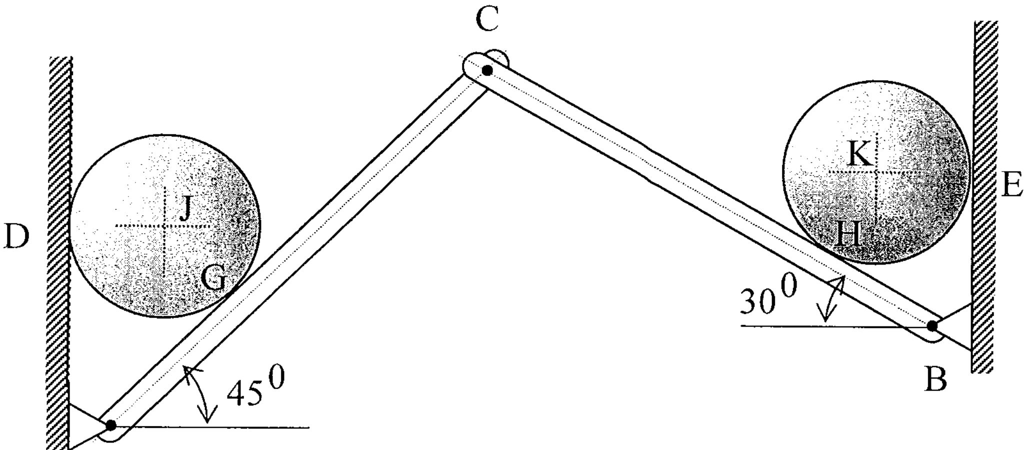
$$\Rightarrow F_{BC} = +100 \text{ kip (tension)}$$

$$\sum F_Y = 0\ (\uparrow\text{+ve}); +R_{AY} + F_{BC} - 100\times\frac{3}{\sqrt{3^2+4^2}} = 0; \Rightarrow R_{AX} = -80 \text{ kip } (\leftarrow)$$

$$\Rightarrow R_{AY} + 100 - 100\times\frac{3}{\sqrt{3^2+4^2}} = 0 \Rightarrow R_{AY} = -40 \text{ kip } (\downarrow)$$

Problem (12): Determine reactions at A, C, D, E, G & H. Assume smooth surface of contact. Given, AB = 3 m, BC = 2 m, sphere J (= radius .60 m & weight 500 gm) & sphere K (= radius .60 m & weight 500 gm)

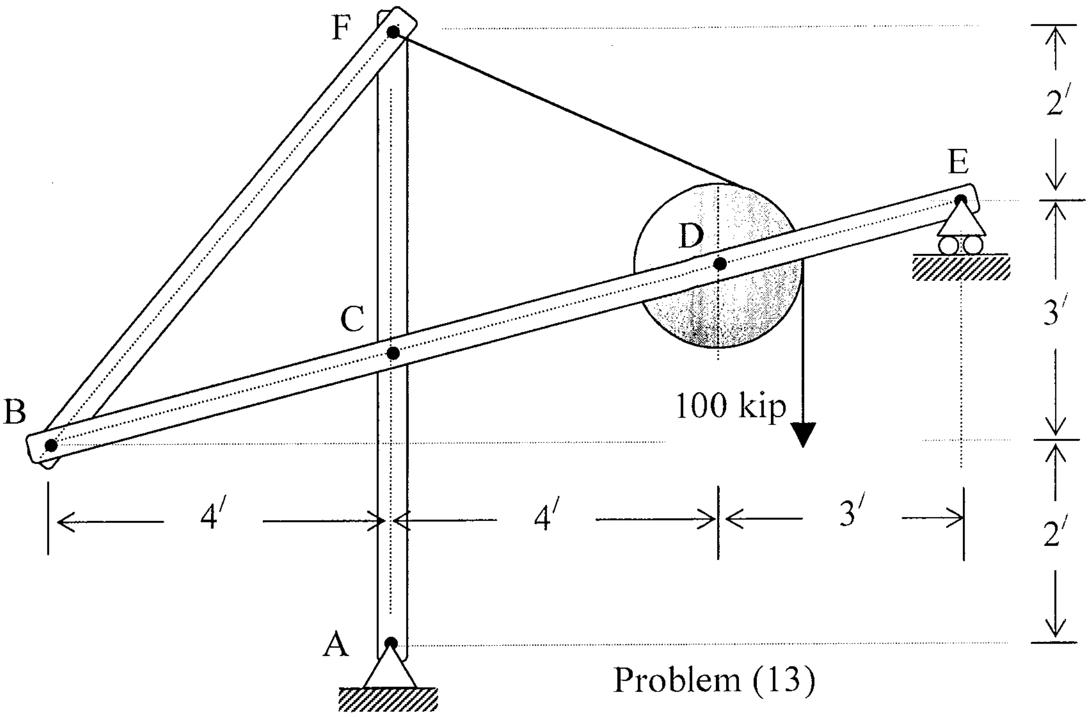
Answer (12): R_{AX} = 92.90 gm; R_{AY} = 609.80 gm; R_{CX} = 303.90 gm; R_{CY} = 390.30 gm; R_D = 500 gm (→); R_E = 288.80 gm (←); R_G = 707 gm & R_H = 577.50 gm.



Problem (12)

Problem (13): Determine reactions at A, E & force in member BF. Given, diameter of pulley 2 feet & its weight 10 kip.

Answer (13): R_{AX} = 0 kip; R_{AY} = 32.85714285 kip (↑); R_{BY} = 77.14285714 kip (↑); F_{BC} = 129.998365 kip (tension).



Problem (13)

Problem (14) – (20): Determine support reactions at A & B and all other internal pins. Identify two-force members & determine the forces in those members. Assume all the members of the framework weightless.

Answer (14): BD, CD & BC are two-force members; R_{AX} = 126.6666667 lb. (←); R_{AY} = 112 lb. (↑); R_{BX} = 246.6666667 lb. (→); R_{BY} = 148 lb. (↑); F_{BD} = 287.6602935 lb. (compression); F_{CD} = 100 lb. (tension) & F_{BC} = 0 lb.

Answer (15): AD, BD, CD & BC are two-force members; R_{AX} = 333.3333333 lb. (←); R_{AY} = 0 lb.; R_{BX} = 333.3333333 lb. (→); R_{BY} = 200 lb. (↑); F_{AD} = 333.3333333 lb. (tension); F_{BD} = 388.7301263 lb. (compression); F_{CD} = 200 lb. (tension) & F_{BC} = 0 lb.

Answer (16): CD & DE are two-force members; R_{AX} = 126.6666667 lb. (←); R_{AY} = 112 lb. (↑); R_{BX} = 246.6666667 lb. (→); R_{BY} = 148 lb. (↑); F_{CD} = 248 lb. (tension); F_{DE} = 385.3056507 lb. (compression).

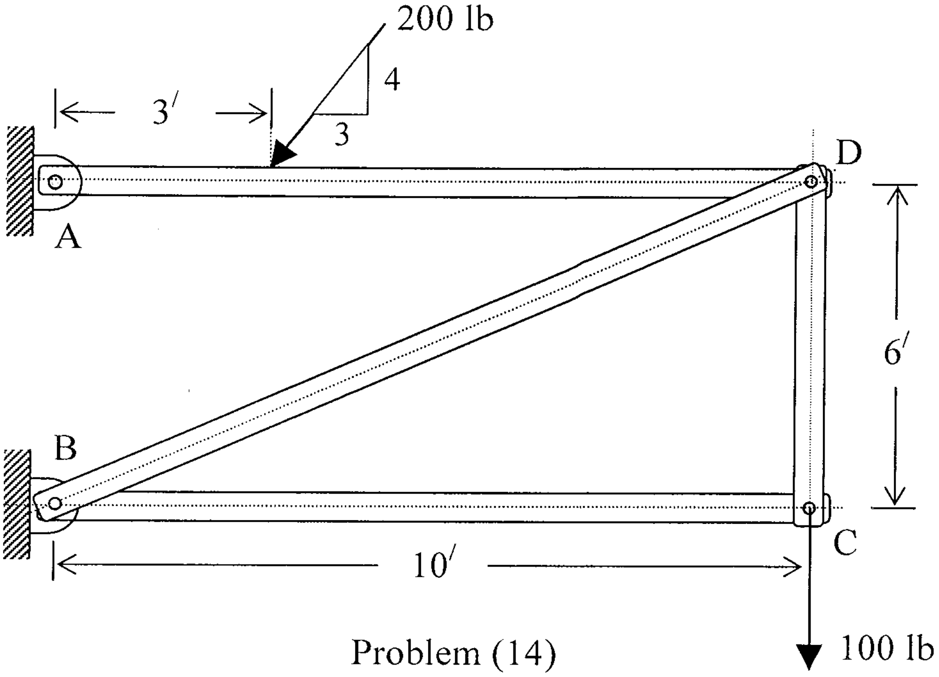
Answer (17): AD, CD & DE are two-force members; R_{AX} = 250 lb. (←); R_{AY} = 0 lb. (↑); R_{BX} = 250 lb. (→); R_{BY} = 150 lb. (↑); F_{AD} = 250 lb. (tension); F_{CD} = 300 lb. (tension); F_{DE} = 390.5124838 lb. (compression).

Answer (18): CD & EF are two-force members; R_{AX} = 126.6666667 lb. (←); R_{AY} = 36 lb. (↓); R_{BX} = 246.6666667 lb. (→); R_{BY} = 296 lb. (↑); F_{CD} = 297.3333333 lb. (tension); F_{EF} = 551.5634345 lb. (compression).

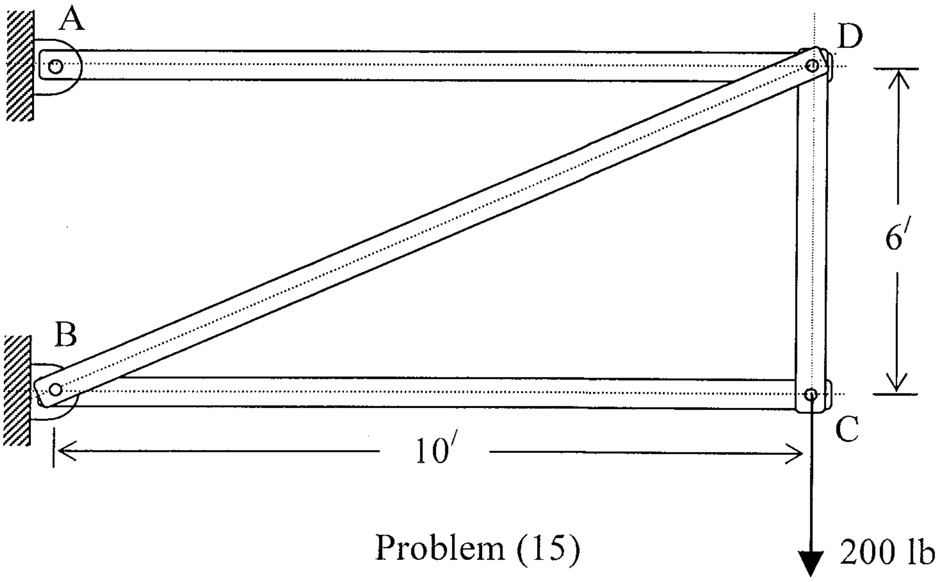
Answer (19): AD, CD & AE are two-force members; R_{AX} = 246.6666667 lb. (←); R_{AY} = 370 lb. (↑); R_{BX} = 366.6666667 lb. (→); R_{BY} = 110 lb. (↓); F_{AD} = 0 lb.; F_{CD} = 0 lb. & F_{AE} = 444.6846573 lb. (tension).

Answer (20): R_{AY} = 100 lb. (↓); R_{BX} = 0 lb.; R_{BY} = 600 lb. (↑); C_X = 166.6666667 lb.; C_Y = 833.3333333 lb.; D_X = 166.6666667 lb.; D_Y = 233.3333333 lb.; E_X = 166.6666667 lb. & E_Y = 333.3333333 lb.

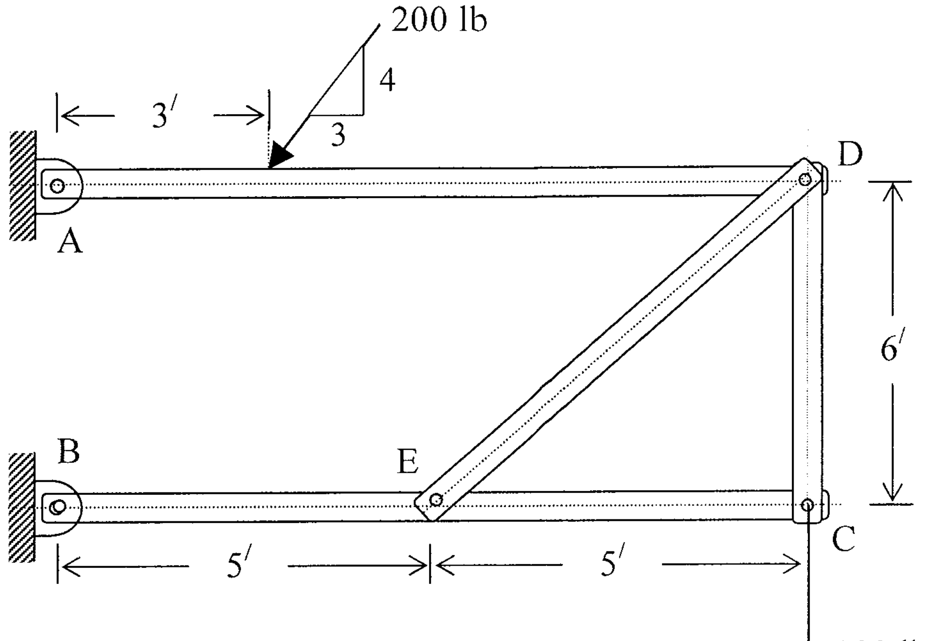
Answer (21): R_{AX} = 533.3333333 lb. (←); R_{AY} = 150 lb. (↑); R_{BX} = 533.3333333 lb. (→); R_{BY} = 50 lb. (↑); C_X = 533.3333333 lb.; C_Y = 350 lb.; D_X = 0 lb.; D_Y = 400 lb. & E_Y = 200 lb. (= tension in the cable).



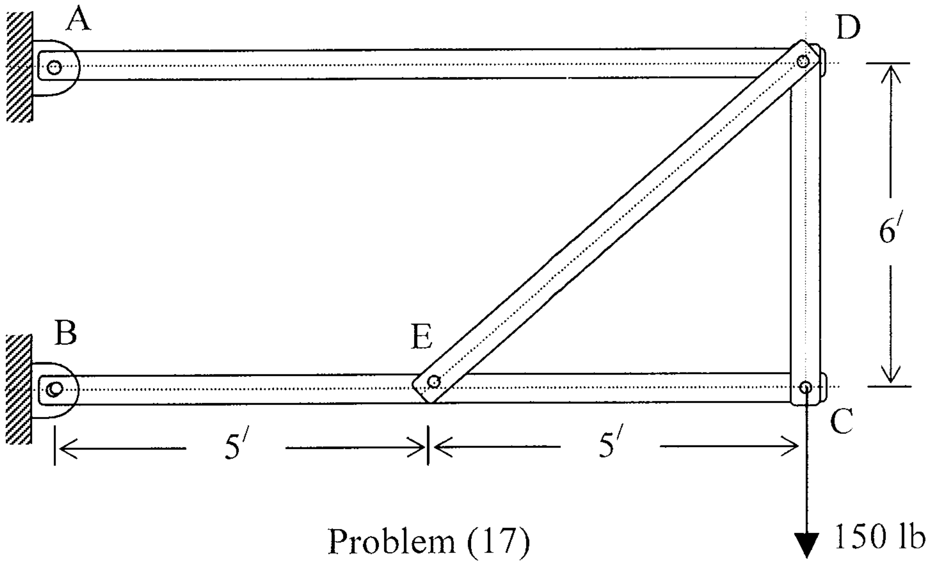
Problem (14)



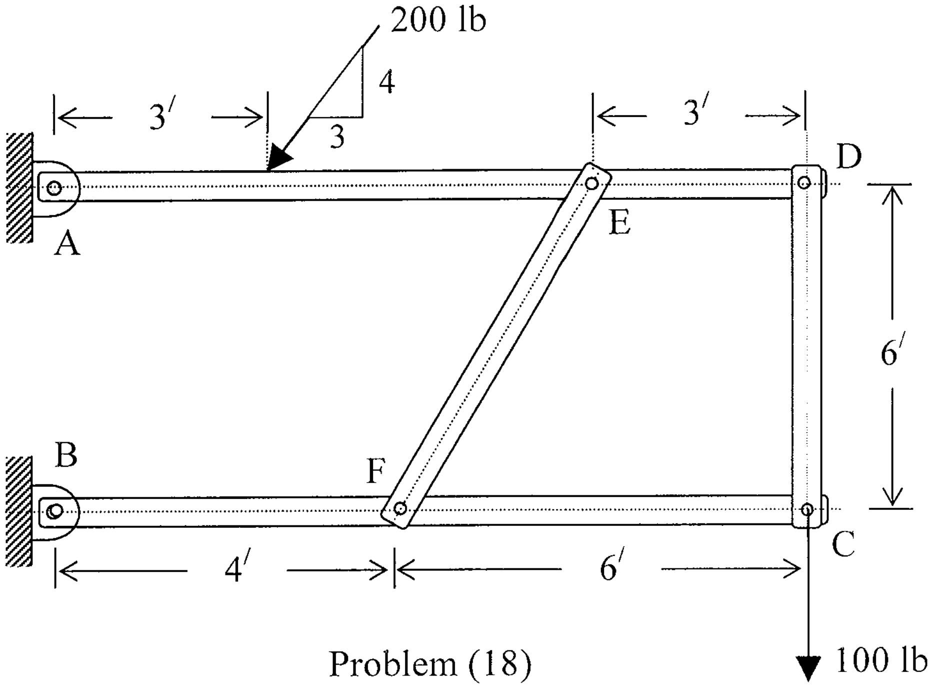
Problem (15)



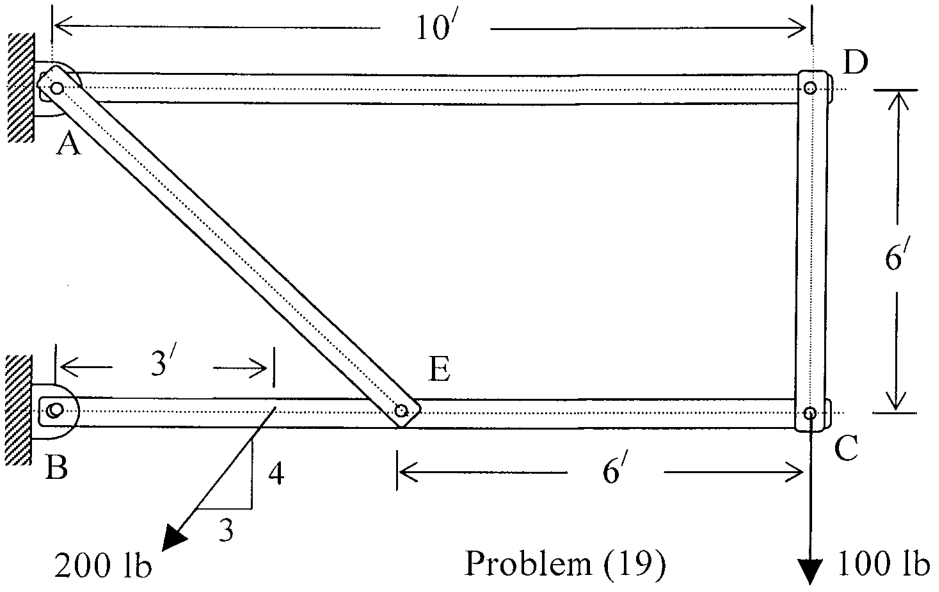
Problem (16)



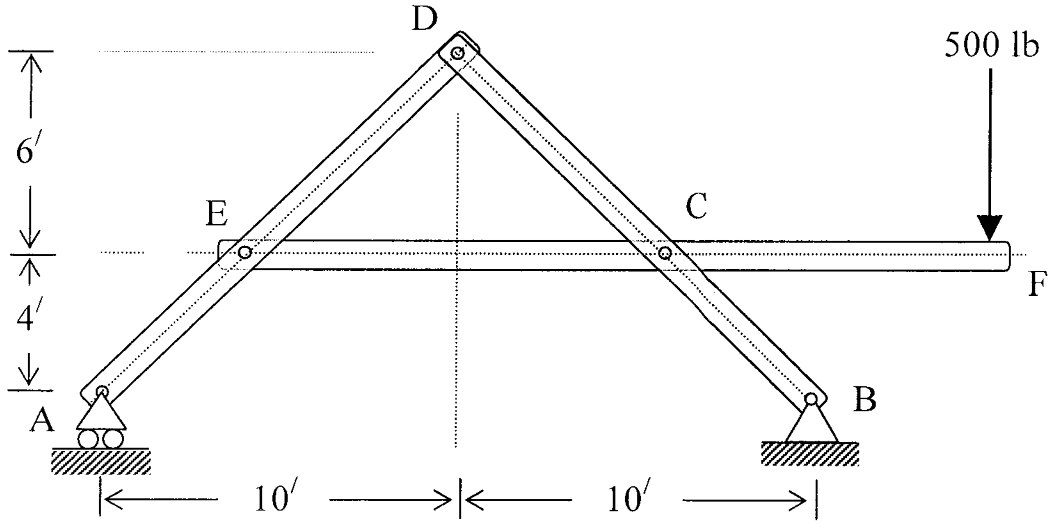
Problem (17)



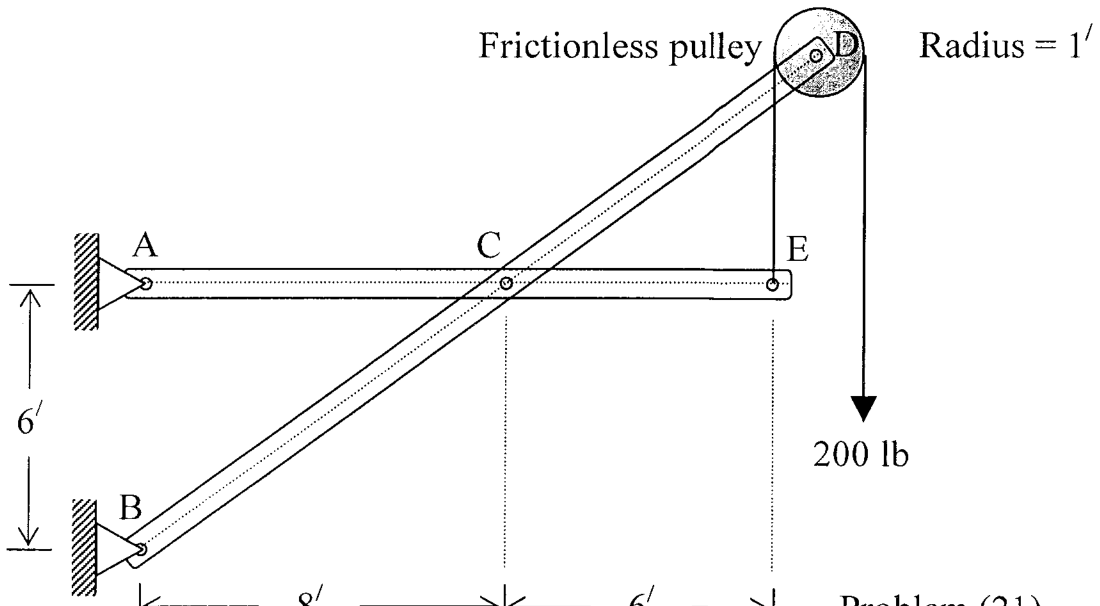
Problem (18)



Problem (19)



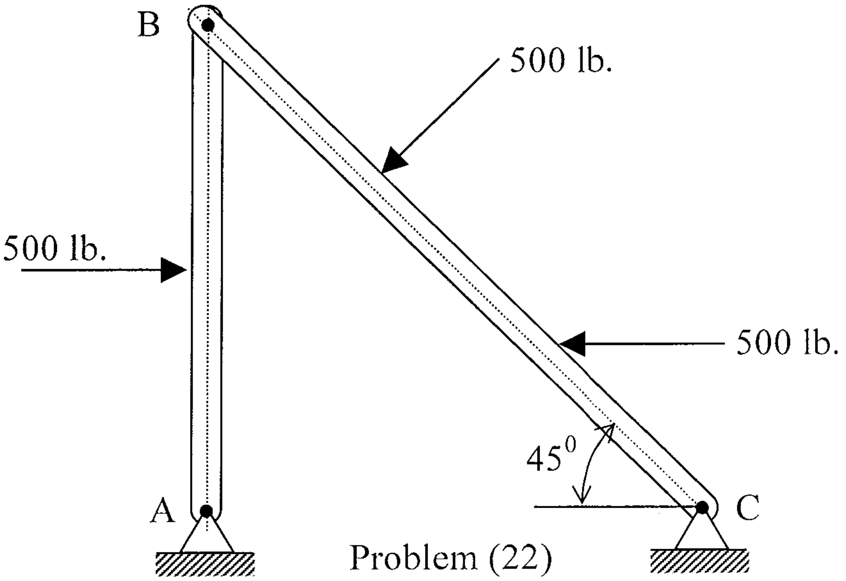
Problem (20)



Problem (21)

Problem (22): A two-bar linkage is loaded with three equal forces of 500 lb, one in the center of one of the bar, the other two at the third-length points of the other bar. Determine the reaction on the ground supports and the force transmitted by the central hinge.

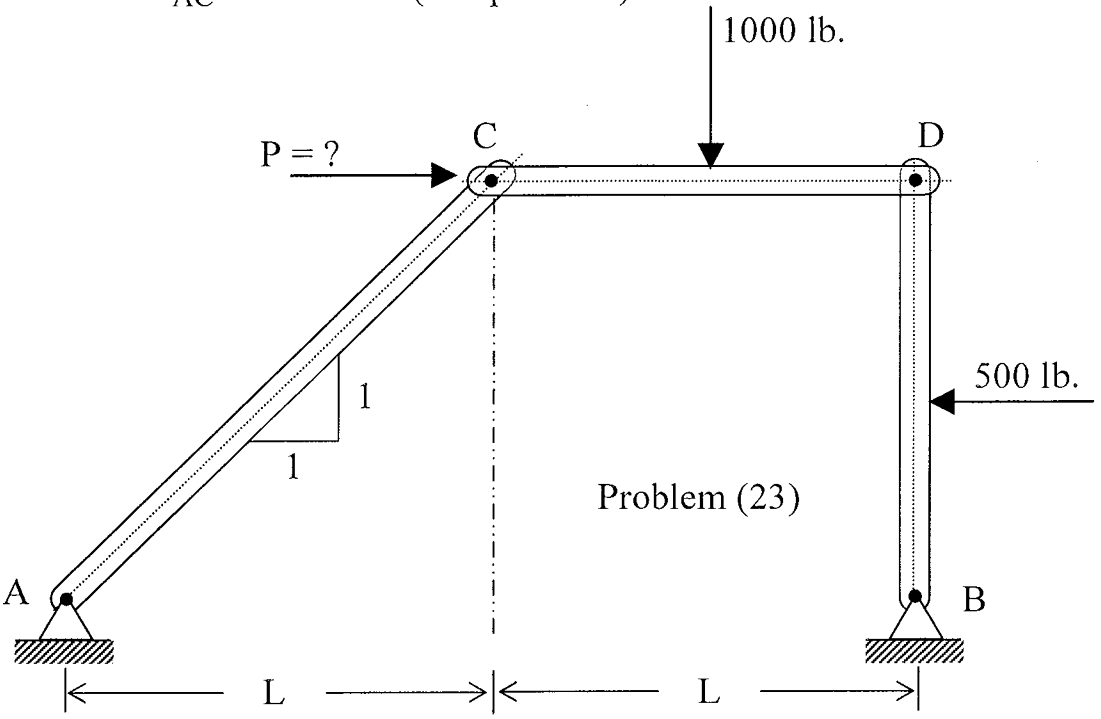
Answer (22): R_{AX} = 250 lb. (←); R_{AY} = 388.0711875 lb. (↑); R_{BX} = 603.5533906 lb. (→); R_{BY} = 34.51779686 lb. (↓); R_{CX} = 250 lb.; R_{CY} = 388.0711875 lb.



Problem (22)

Problem (23): Two of the bars of three three-bar linkage are subjected to forces at their mid points. What is the force P required for equilibrium? What are the ground support reactions, internal hinge reactions & force in member AC?

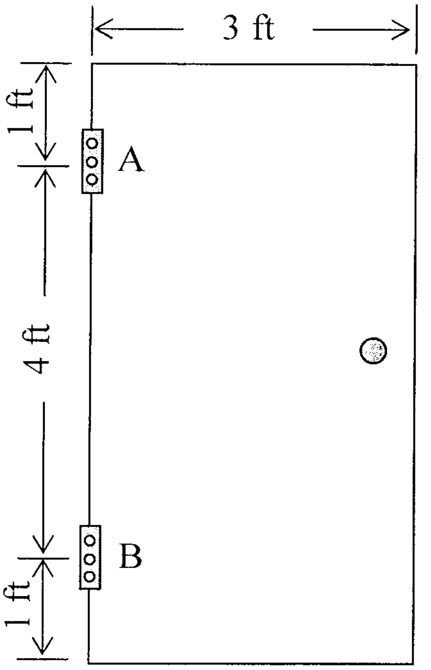
Answer (23): P = 250 lb. (←); R_{AX} = 500 lb. (→); R_{AY} = 500 lb. (↑); R_{BX} = 250 lb. (→); R_{BY} = 500 lb. (↑); R_{CX} = 250 lb.; R_{CY} = 500 lb.; R_{DX} = 250 lb.; R_{DY} = 500 lb. & $F_{AC} = 500\sqrt{2}$ lb. (compression).



Problem (23)

Problem (24): A door of dimensions 3 ft by 6 ft is hung on two hinges, 4 ft apart and 1 ft from top and bottom. The door weighs 30 lb., and its weight is uniformly distributed over its area. Determine the forces exerted by the door on the hinges.

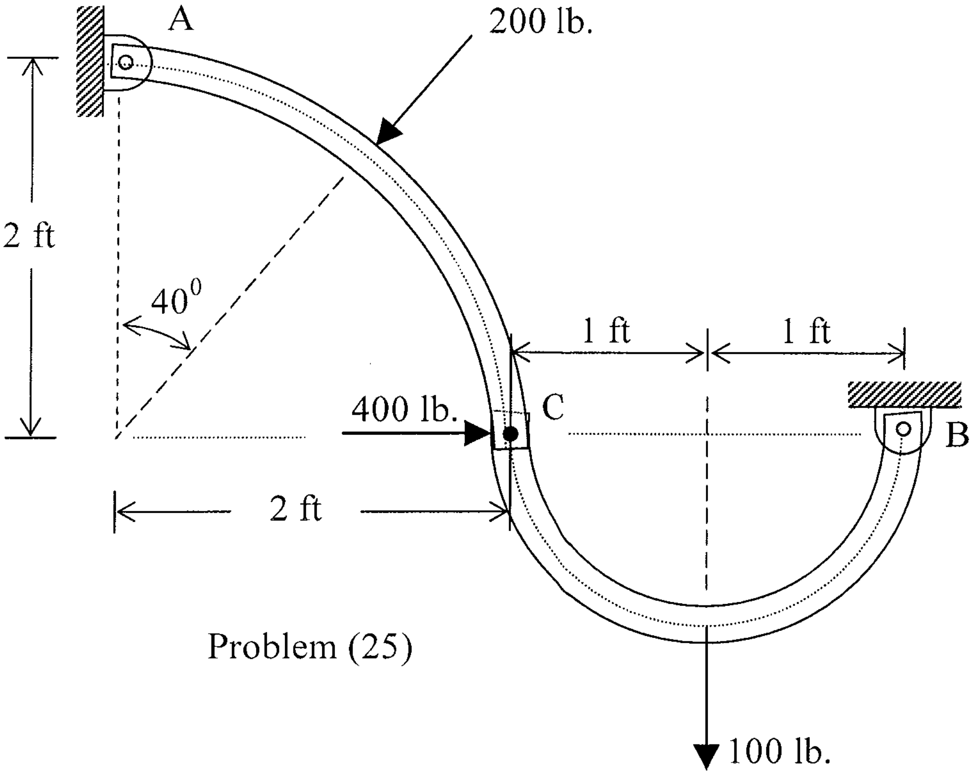
Answer (24): R_{AX} = 11.25 lb. (←); R_{BX} = 11.25 lb. (→) & $R_{AY} + R_{BY}$ = 30 lb. (↑)



Problem (24)

Problem (25): A structure consists of two circularly curved beams hinged at A, C & B and is loaded with forces of 100, 200 & 400 lb. Find the reactions at A< B & C.

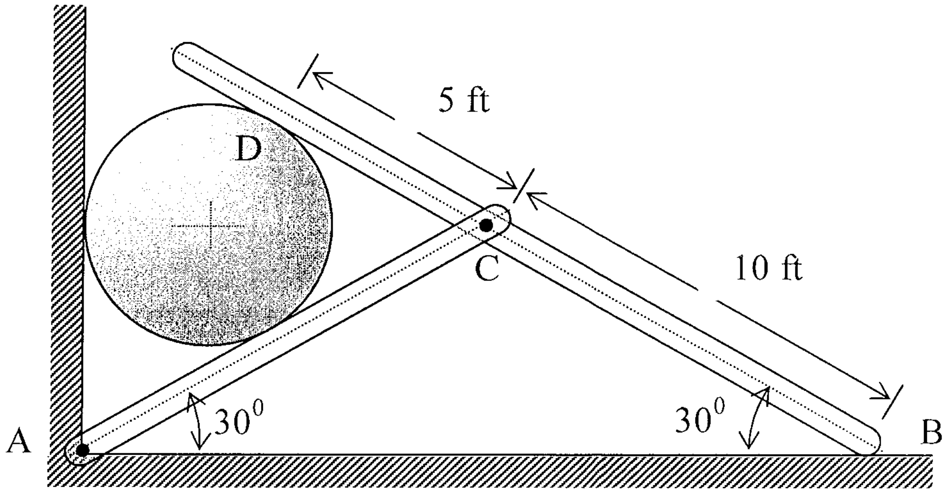
Answer (25): R_{AX} = 50 lb. (←); R_{AY} = 203.2088886 lb. (↑); R_{BX} = 221.4424781 lb. (←); R_{BY} = 50 lb. (↓); R_{CX} = 221.4424781 lb.; R_{CY} = 50 lb.



Problem (25)

Problem (26): The bar BCD is pinned to the bar AC at C and rests on a smooth surface plane at B. the smooth cylinder weighs 500 lb. determine the force at pin C acting on BCD.

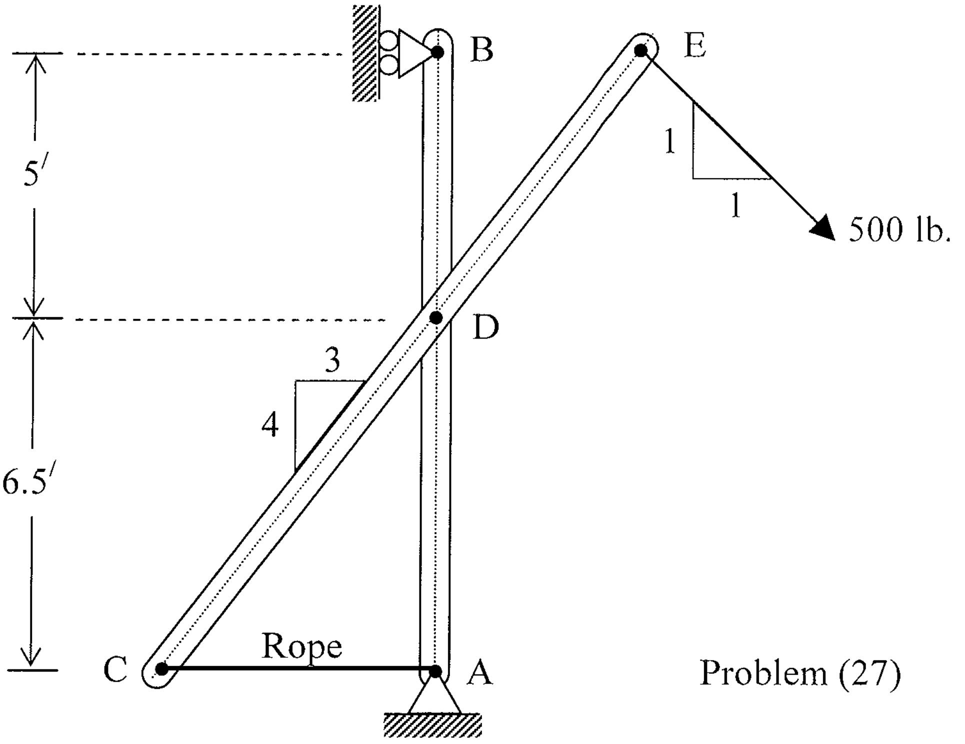
Answer (27): $R_{CX} = 288$ lb. (←) & $R_{CY} = 833$ lb. (↓)



Problem (26)

Problem (27): Determine tension in the rope AC.

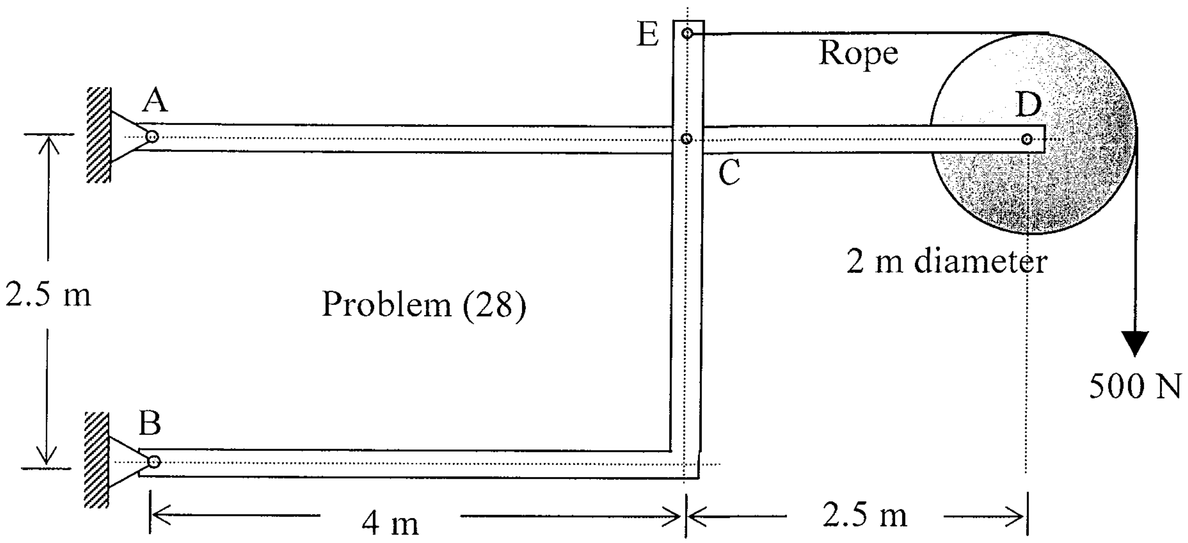
Answer (27): $F_{AC} = 475.9372566$ lb. (tension)



Problem (27)

Problem (28): Determine support reactions at A & B and internal pin reactions at C & D. Also determine the tension in the rope.

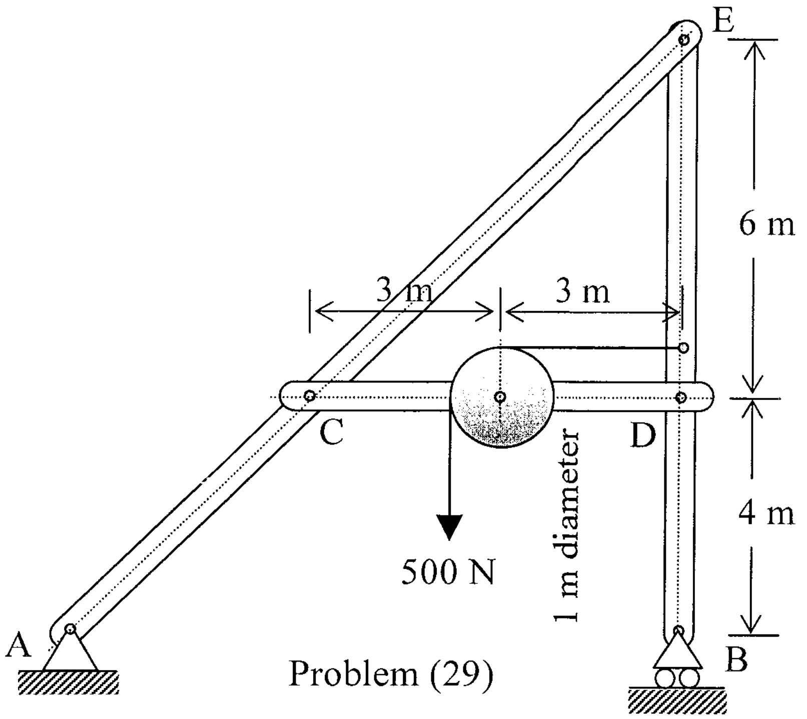
Answer (28): $R_{AX} = 1300$ N (←), $R_{AY} = 625$ N (↑), $R_{BX} = 1300$ N (→), $R_{BY} = 125$ N (↓), $C_X = 1800$ N, $C_Y = 125$ N, $D_X = 500$ N, $D_Y = 500$ N & tension in the rope = 500 N.



Problem (28)

Problem (29): Determine support reactions at A & B and internal pin reactions at C, D & E.

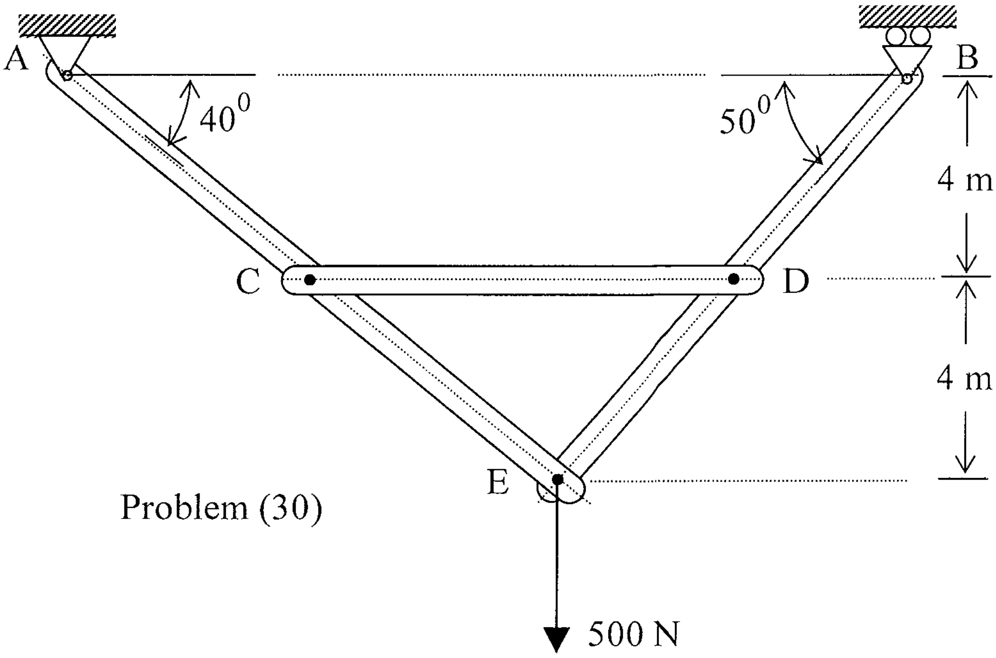
Answer (29): $R_{AX} = 0$ N, $R_{AY} = 175$ N (↑), $R_{BY} = 325$ N (↑), $C_X = 41.66666667$ N, $C_Y = 250$ N, $D_X = 458.3333333$ N, $D_Y = 250$ N, $E_X = 41.66666667$ N, $E_Y = 75$ N.



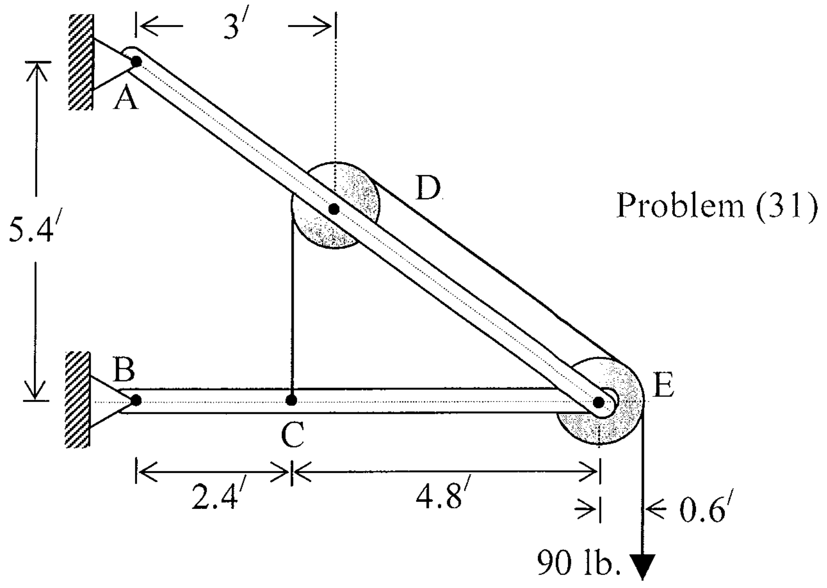
Problem (29)

Problem (30): Determine support reaction at A & B and force in the member CD.

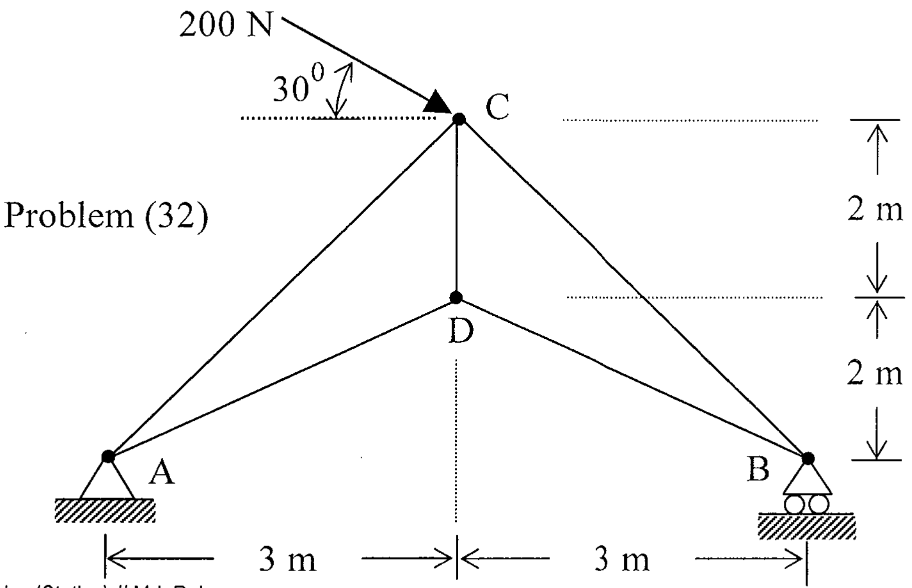
Answer (30): $R_{AX} = 0$ N, $R_{AY} = 206.5879556$ N (↑), $R_{BY} = 293.4120444$ N (↑) & $F_{CD} = 492.4038765$ N (compression)


Problem (30)

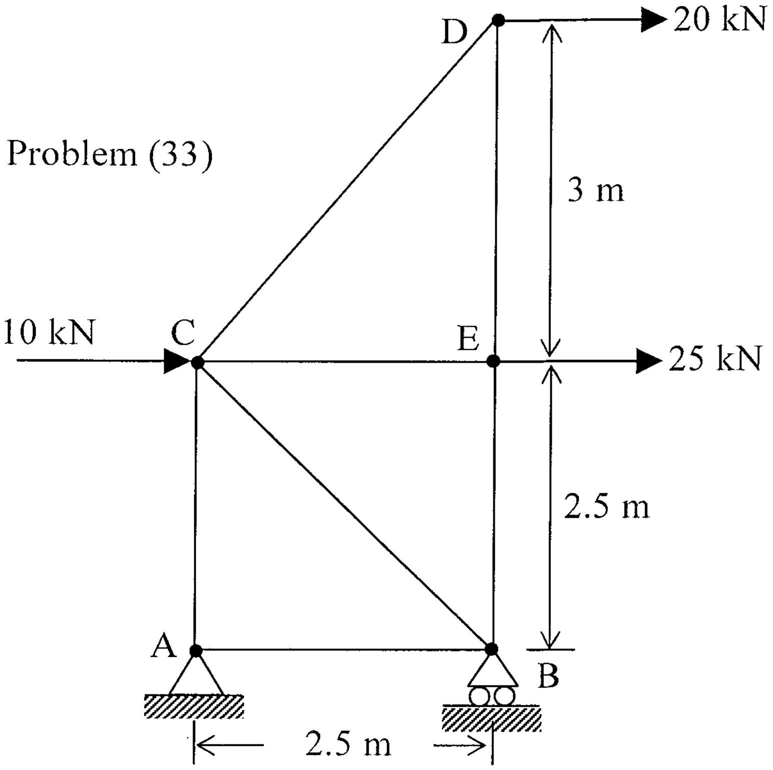
Problem (31): Determine support reaction at A & B.


Problem (31)

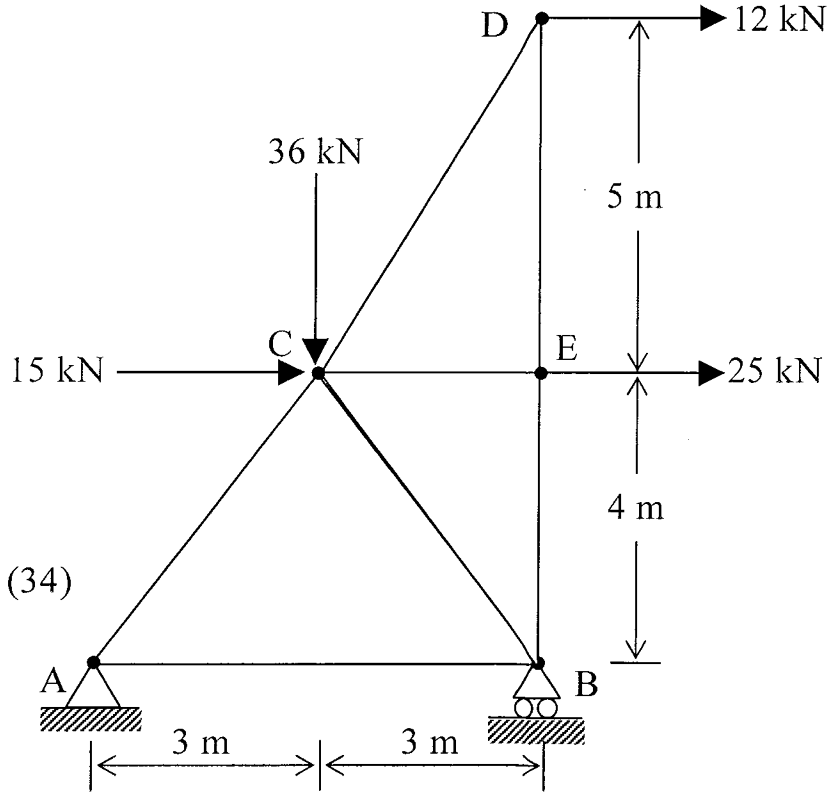
Problem (32) – (37): For the trusses, Determine all the bar forces.

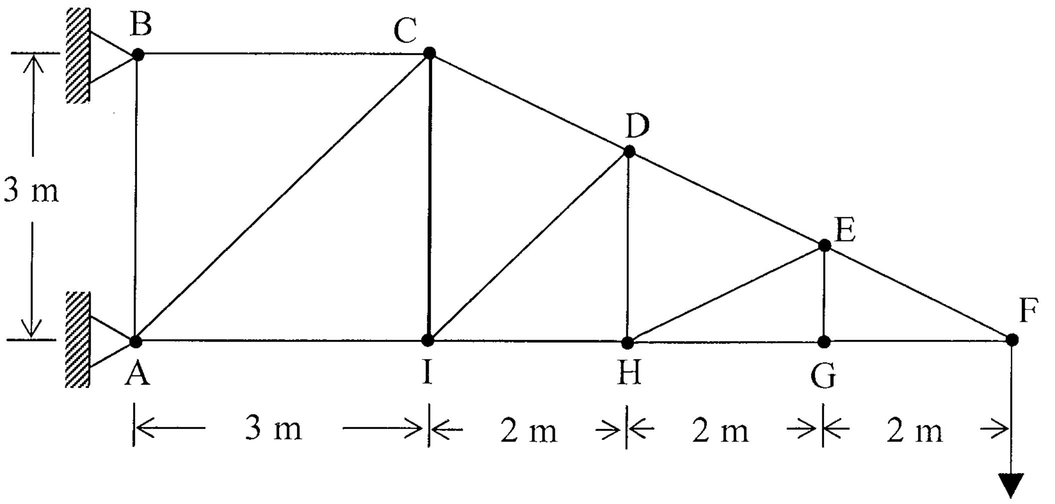

Problem (32)

Problem (33)

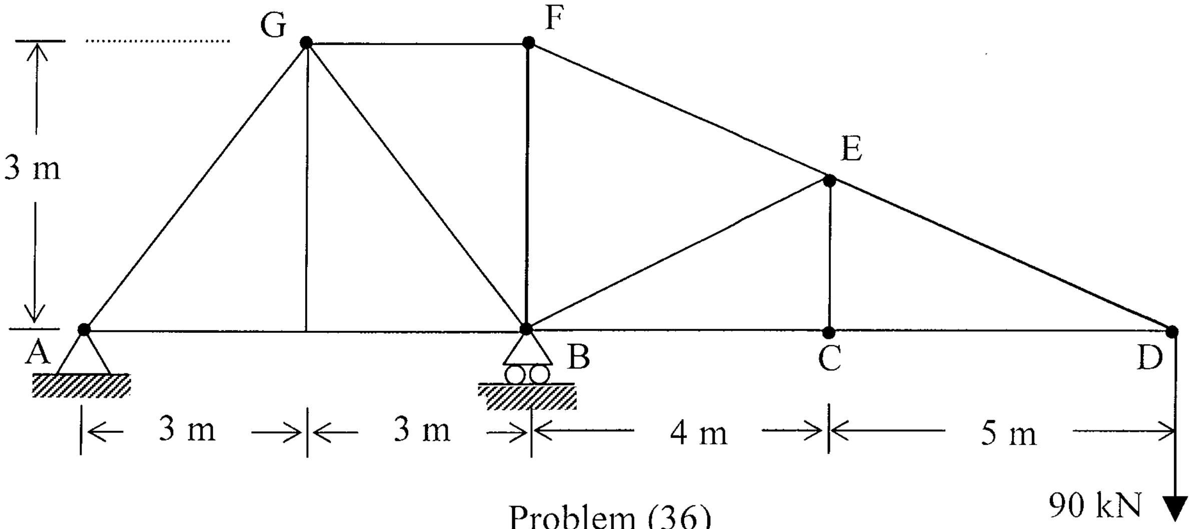


Problem (34)



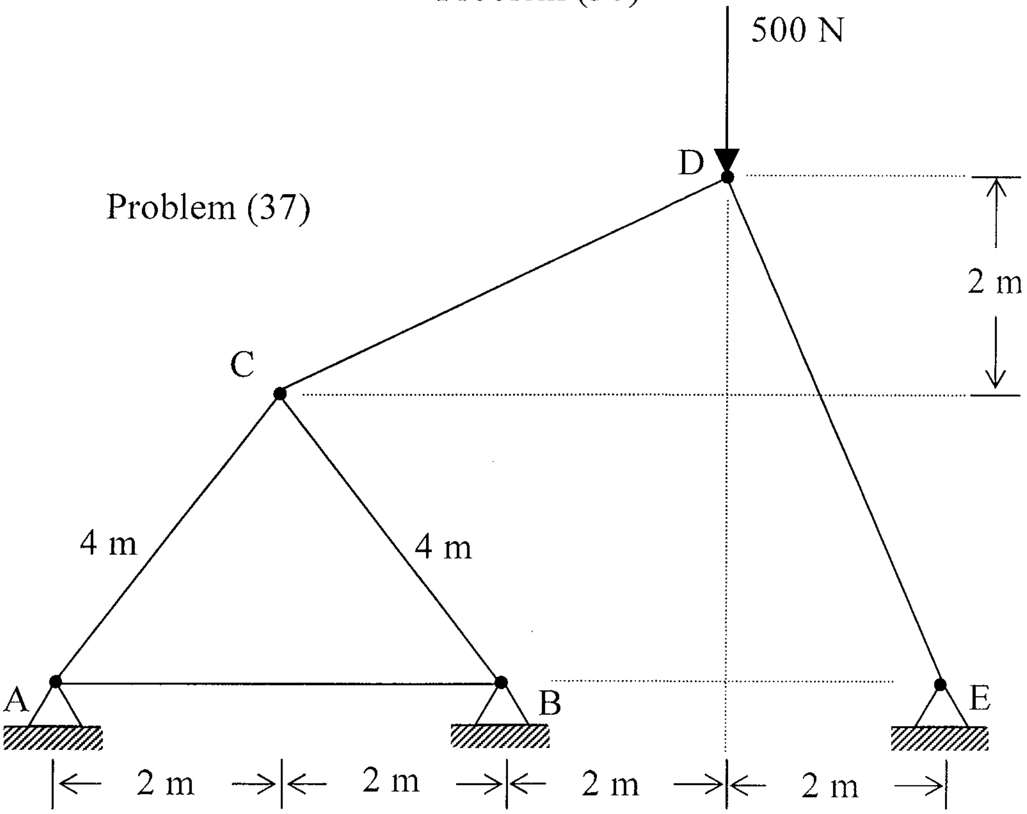


Problem (35)



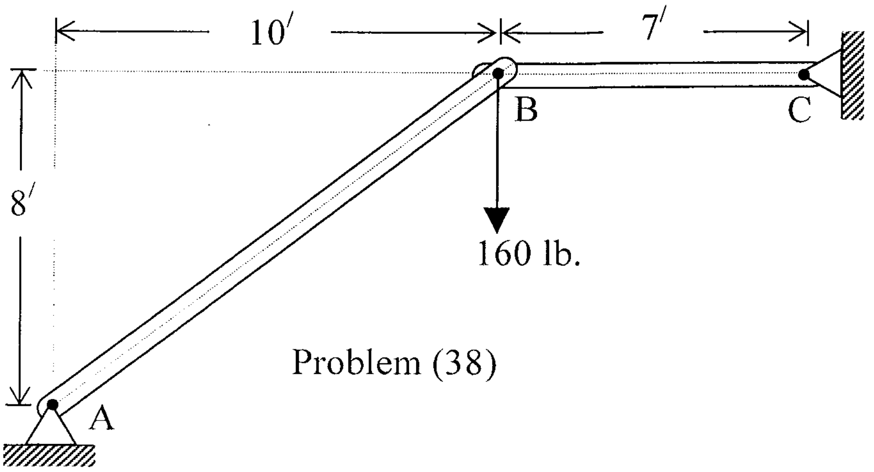
Problem (36)

Problem (37)



Problem (38): Determine the force in the member AB & BC.

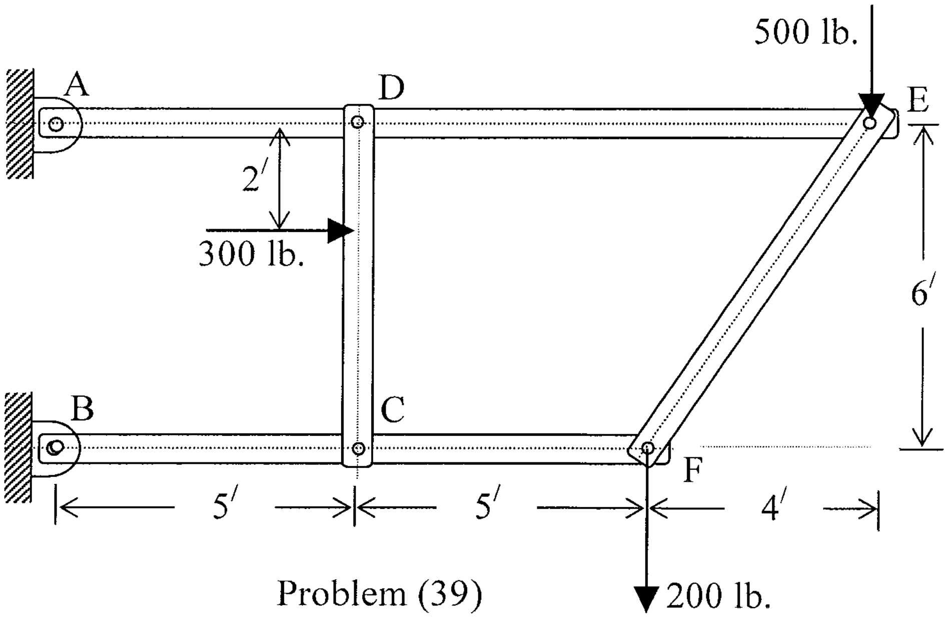
Answer (38): F_{AB} = 256.1249695 lb. (compression); F_{BC} = 200 lb. (compression)



Problem (38)

Problem (39): The cantilever frame consists of two horizontal members AE and BF, a vertical member CD and an inclined member EF. Calculate the components of pin reaction at A and force in the member EF. All the members have been assumed to be weightless.

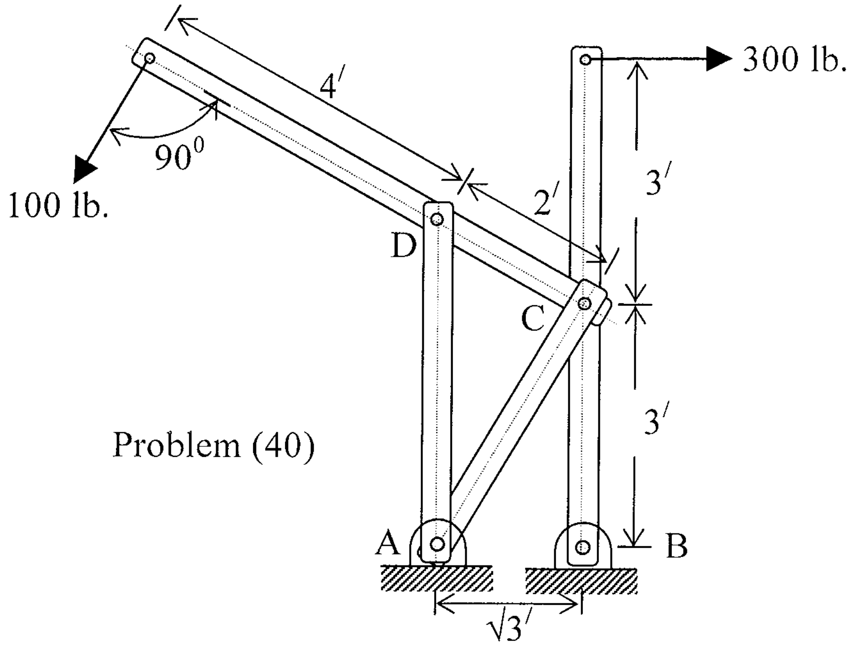
Answer (39): R_{AX} = 1700 lb. (←); R_{AY} = 1350 lb. (↓); F_{EF} = 2704.163457 lb. (compression).



Problem (39)

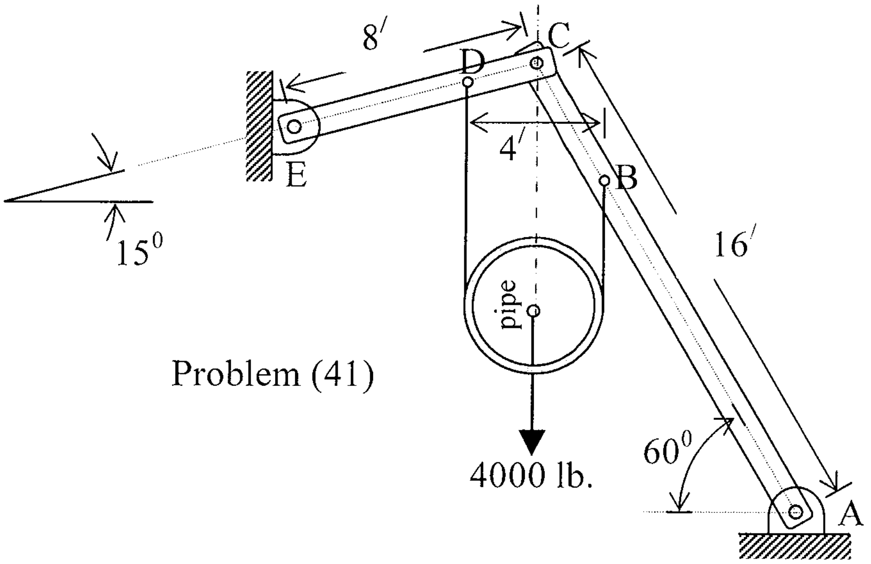
Problem (40): Determine pin reactions at A & B and forces in member AC & AD?

Answer (40): $R_{AX} = 550$ lb. (←); $R_{AY} = 350\sqrt{3}$ lb. (↓); $R_{BX} = 300$ lb. (→); $R_{BY} = 400\sqrt{3}$ lb. (↑); $F_{AC} = 1100$ lb. (tension) & $F_{AD} = 200\sqrt{3}$ lb. (compression).



Problem (40)

Problem (41): The pipe load is 4000 lb. on the vertical center line of pin C. Which pin, A, C, E, has the maximum load on it? How much this load?



Problem (41)

Answer (41):

Pin	Reaction components (lb.)	Resultant reaction (ib.)
at A	R_{AX} = 1491.180955 lb. (←)	3424.51219 lb.
	R_{AY} = 3082.80117 lb. (↑)	
at C	R_{CX} = 1491.180955 lb.	1842.845362 lb.
	R_{CY} = 1082.80117 lb.	
at E	R_{EX} = 1491.180955 lb. (←)	1750.678246 lb.
	R_{EY} = 917.1988229 lb. (↑)	
Maximum load will taken by pin at ‘A’		

Chapter Six Non-Coplanar Forces

Three Rectangular Components of a Force:

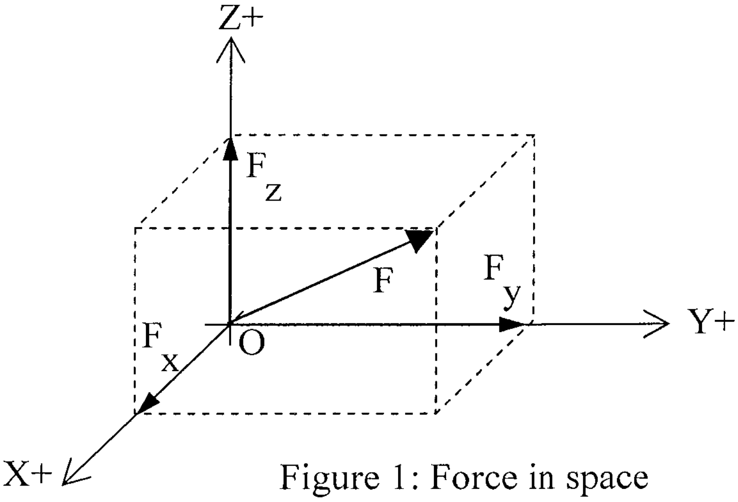


Figure 1: Force in space

F_x = Component of force F in X-direction

F_y = Component of force F in Y-direction

F_z = Component of force F in Z-direction

α = angle between F_x and F.

β = angle between F_y and F.

γ = angle between F_z and F.

$$F_x = F\cos\alpha\,;\ \cos\alpha = \frac{F_x}{F}$$

$$F_y = F\cos\beta\,;\ \cos\beta = \frac{F_y}{F}$$

$$F_z = F\cos\gamma\,;\ \cos\gamma = \frac{F_z}{F}$$

$$(F_x)^2 + (F_y)^2 + (F_z)^2 = (F\cos\alpha)^2 + (F\cos\beta)^2 + (F\cos\gamma)^2$$

$$= F^2\,[\cos^2\alpha + \cos^2\beta + \cos^2\gamma] = F^2$$

Since, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$F = \sqrt{(F_x)^2 + (F_y)^2 + (F_z)^2}$$

Concurrent Forces in Space:

$$F = \sqrt{(\sum F_x)^2 + (\sum F_y)^2 + (\sum F_z)^2}$$

$$\cos\theta_x = \frac{\sum F_x}{F};\ \cos\theta_y = \frac{\sum F_y}{F}\ \&\ \cos\theta_z = \frac{\sum F_z}{F}$$

θ_x, θ_y & θ_z are the angles made by component forces $\sum F_x$, $\sum F_y$ & $\sum F_z$ with the resultant force F respectively.

Definition of a Force in Space: For convenience, we shall sometimes define a force by a sequence of four numbers in the order of F, x, y, z. Thus, the number (500, -6, 4, 8) means that the force F = 500 lb. passes through the origin and the point x = - 6. y = 4 and z = 8. If the line of action does not pass through the origin, it would take seven numbers to define a force according to this scheme $(F, x_1, y_1, z_1, x_2, y_2, z_2)$ where subscript 1 represents tail co-ordinate & subscript 2 represents head co-ordinate of force vector.

Equilibrium of Concurrent Forces in Space: Summing the forces in X, Y & Z directions -

$$\sum F_x = 0,\ \sum F_y = 0\ \&\ \sum F_z = 0$$

Non-coplanar Parallel Forces: In the case on non-coplanar, parallel forces, it is convenient to use co-ordinate planes such that one of them is perpendicular to the lines of action of the forces (see figure 2).

F = resultant of F_1, F_2, F_3 & F_4 (parallel forces)

$(\bar{x}, \bar{y})$ = location of resultant force F from Y- & X-axis respectively

(x_1, y_1) = location of component force F_1 from Y- & X-axis respectively

(x_2, y_2) = location of component force F_2 from Y- & X-axis respectively

(x_3, y_3) = location of component force F_3 from Y- & X-axis respectively

(x_4, y_4) = location of component force F_4 from Y- & X-axis respectively

Summing the forces in the Z- direction.

$$\sum F_z = -F_1 - F_2 + F_3 - F_4 = F$$

From which magnitude of resultant force F is found.

Taking moments of the forces about X- & Y-axis respectively

$$\sum M_x = F_1 y_1 - F_2 y_2 + F_3 y_3 - F_4 y_4 = \sum F_z \bar{y} = F\bar{y}$$

$$\sum M_y = F_1 x_1 + F_2 x_2 - F_3 y_3 - F_4 y_4 = \sum F_z \bar{x} = F\bar{x}$$

From which location $(\bar{x}, \bar{y})$ of resultant force F is found.

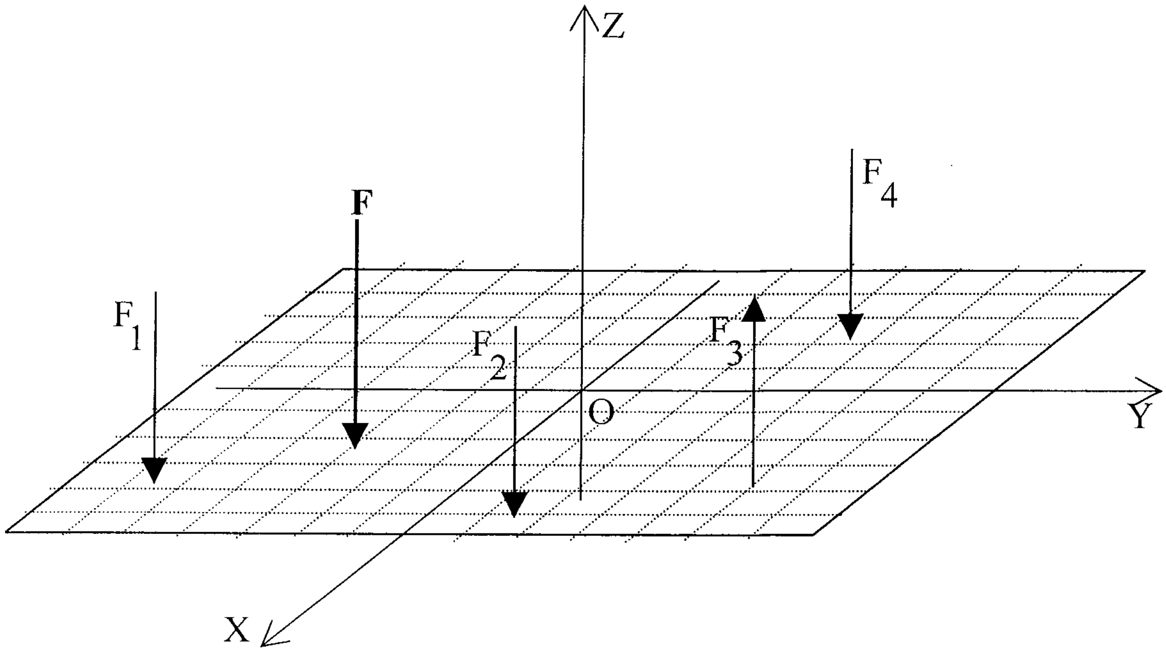


Figure 2: Resultant of Non-Coplanar Parallel Forces

Equilibrium of a General Force System in Space: If a general force system in space is in equilibrium, there must be neither resultant force nor a resultant couple; that is, $\sum F = 0$ & $\sum M = 0$.

$$\sum F_x = 0,\ \sum F_y = 0\ \&\ \sum F_z = 0$$

$$\sum M_x = 0,\ \sum M_y = 0\ \&\ \sum M_z = 0$$

Non-concurrent, Non-parallel Forces in Space:

$$F = \sqrt{(F_x)^2 + (F_y)^2 + (F_z)^2}$$

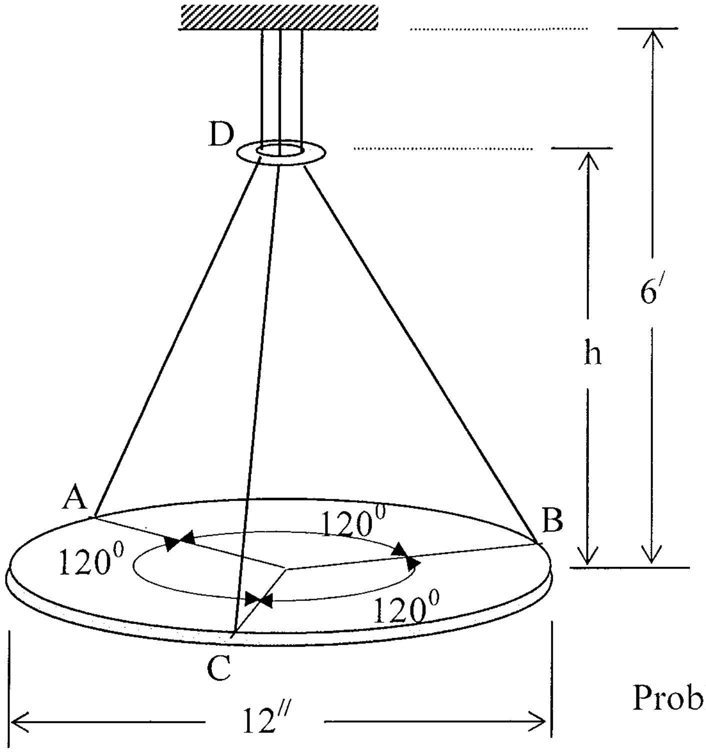
$$M = \sqrt{(M_x)^2 + (M_y)^2 + (M_z)^2}$$

Where, F is the magnitude of the resultant force & M is the magnitude of the resultant moment. If $R = 0$, but $M \neq 0$, the resultant is a couple. If $R \neq 0$, but $M = 0$, the resultant is a force, which may be represented by a force and a couple..

Problems

Problem (1): A uniform 300-lb disk is supported in a horizontal position by 3 chords attached symmetrically. A slip-ring at D may be pushed downward to increase the angle that the cords make with the vertical. If the cords have a safe strength of 150 lb. each, find the minimum safe value of h.

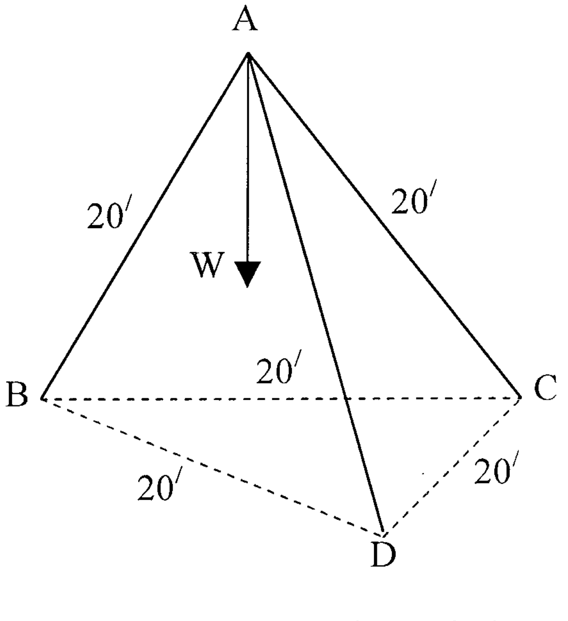
Answer (1): $h \geq 8.485281374$ inch.



Problem (1)

Problem (2): Three timbers, AB, AC & AD, each 20 ft. long, form a tripod. The ends of the timber on the ground form an equilateral triangle BCD, the sides of which are each 20 ft. long. If the safe compressive load for each timber is 20000 lb., what safe load W may be suspended from the point A.

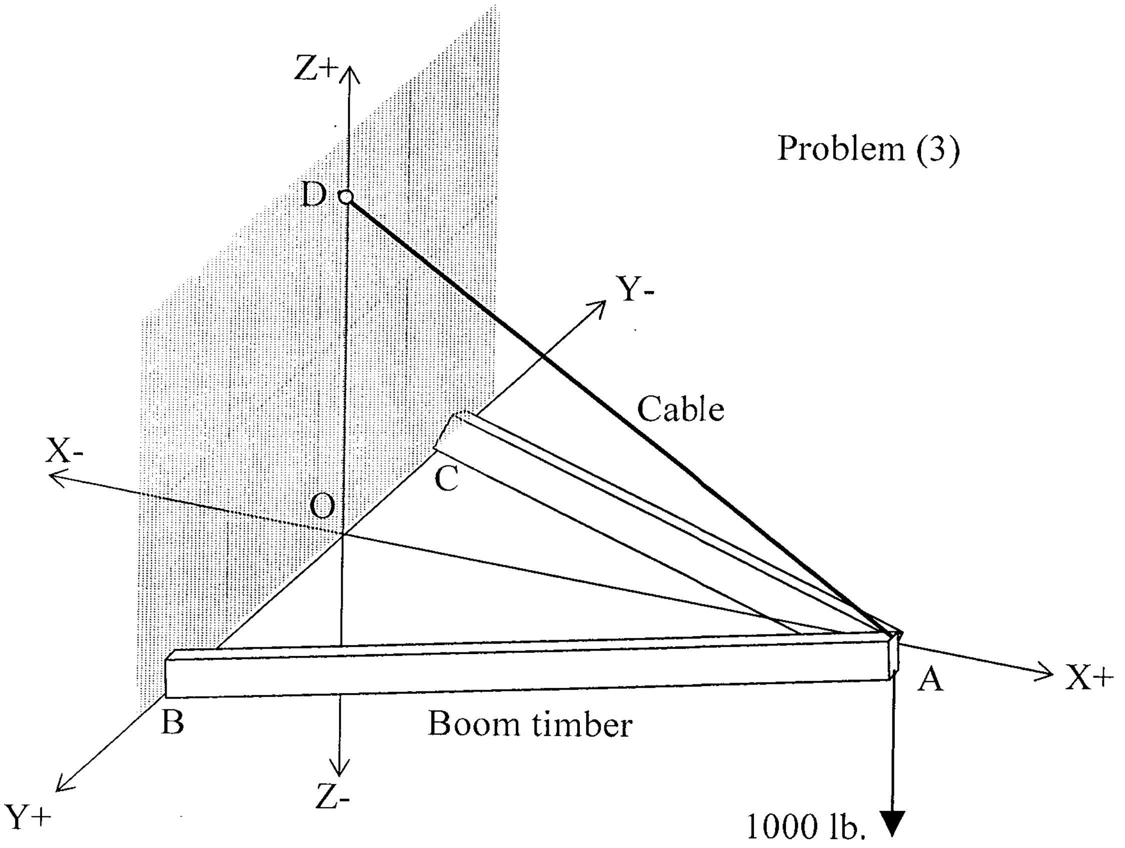
Answer (2): W = 48989.79486 lb. (maximum)



Problem (2)

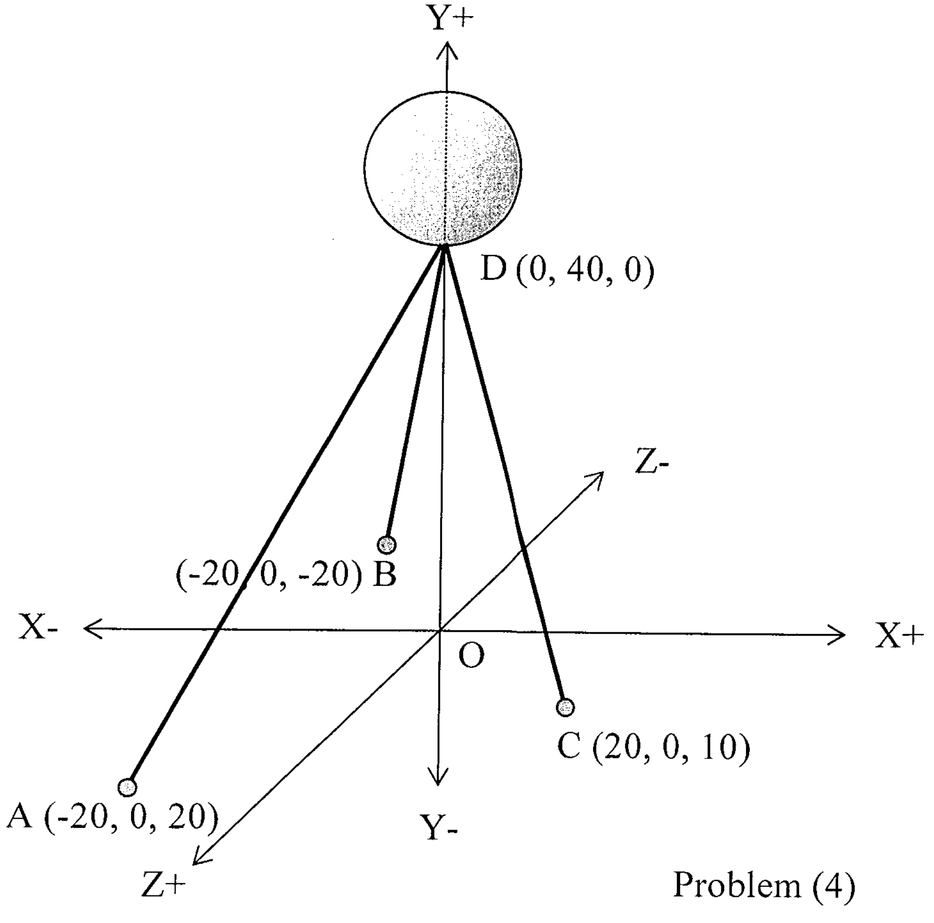
Problem (3): A boom constructed of two timbers AB & AC which are fastened together at A. the cable AD holding the timers in a horizontal plane and supporting a load of 1000 lb. If BO = CO = DO = 6 ft. & OA = 8 ft., find the tension in the cable & the force in each timber.

Answer (3): F_{AD} = 1666.6667 lb. (tension); F_{AB} & F_{AC} = 833.3333 lb. (compression).



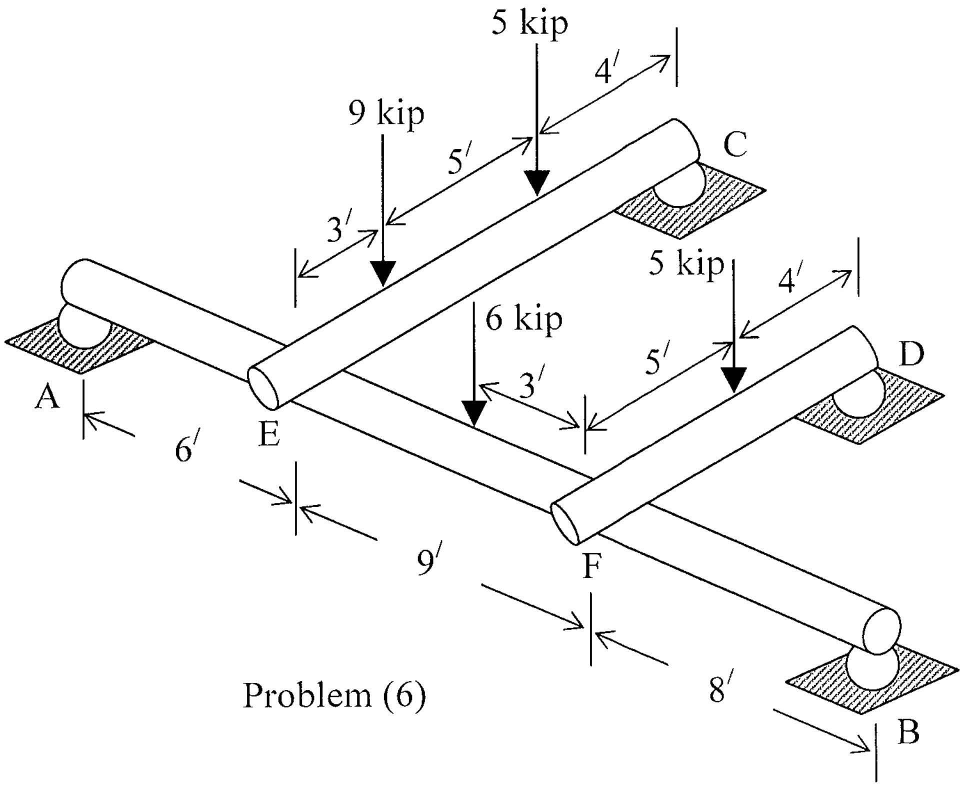
Problem (4): The balloon is held by three mooring cables. If the net lift is 800 lb., what is the force in each cable?

Answer (4): F_{AD} = 122.4744871 lb. (tension); F_{BD} = 367.4264614 lb. (tension) & F_{CD} = 458.2575695 lb. (tension)



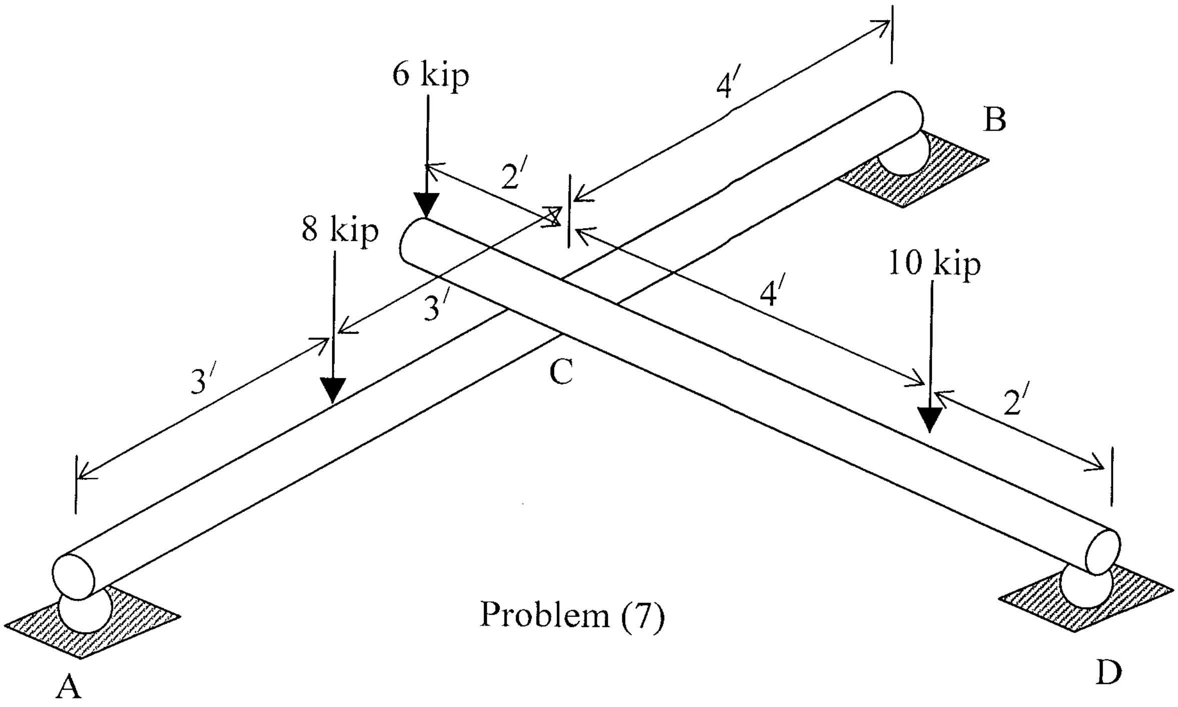
Problem (5): In problem (4), if the maximum load that any one cable can carry is 900 lb., what is the maximum permissible lift? (Note: This does not mean that 900 lb. exist in all cables simultaneously.)
Answer (5): 1571.16881 lb.

Problem (6): Determine the roller reactions at the ends (A & B) of 23-ft beam. All beams are weightless, and all surfaces are smooth.
Answer (6): $R_A = 9.86352654$ kip (↑) & $R_B = 6.775362319$ kip (↑)



Problem (6)

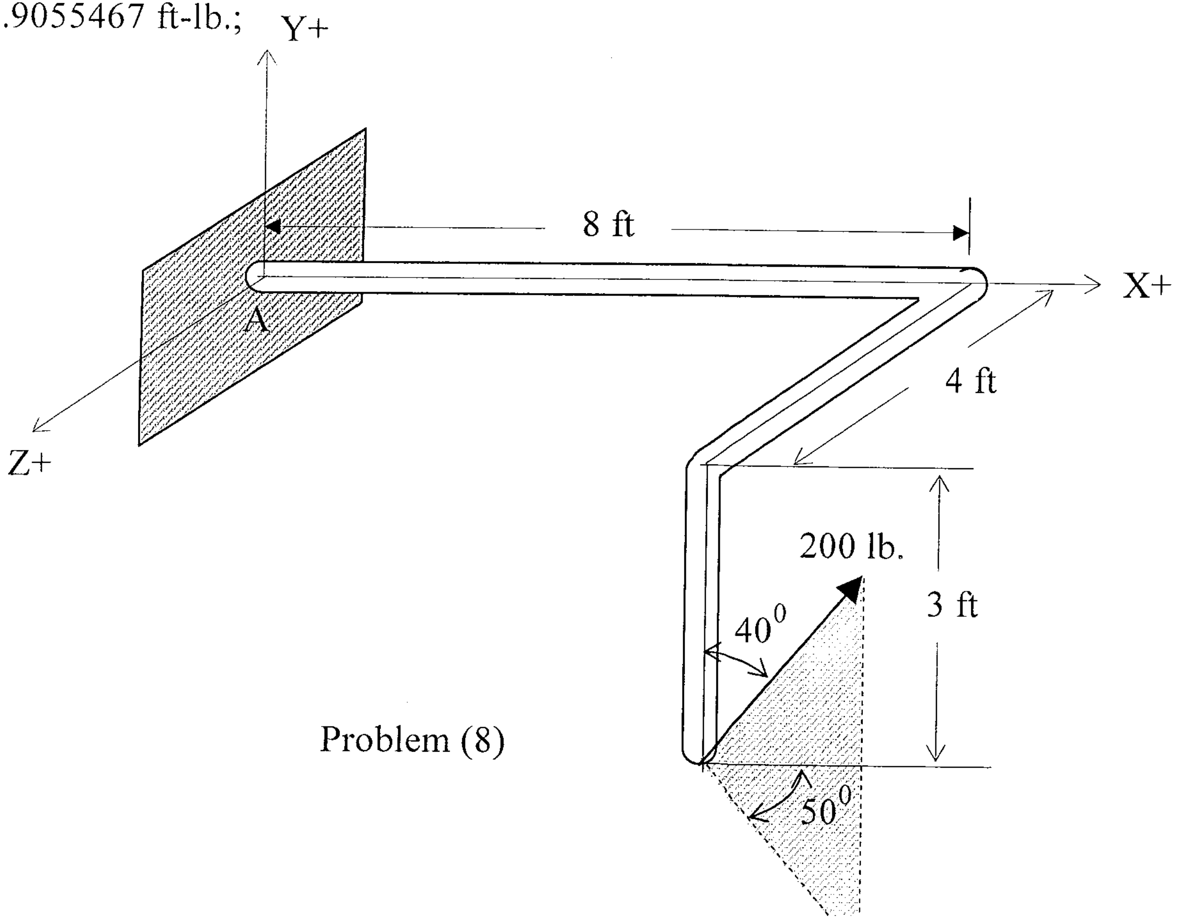
Problem (7): Determine the roller reactions at the ends (A, B & D) & internal vertical reaction at C. All beams are weightless, and all surfaces are smooth.
Answer (7): $R_A = 7.733333333$ kip (↑); $R_B = 5.60$ kip (↑); $R_D = 4.666666667$ kip (↑) & $R_C = 5.333333333$ kip.



Problem (7)

Problem (8): The bar is bent so that its three parts are parallel to the X, Y and Z axes, as shown in the figure. The bar is rigidly attached to the wall at A. the 200-lb force makes an angle of 40^0 with the Y-axis. The vertical plane containing the force makes an angle of 50^0 with the X-axis. Determine the force & moment that the wall must supply at A for equilibrium.

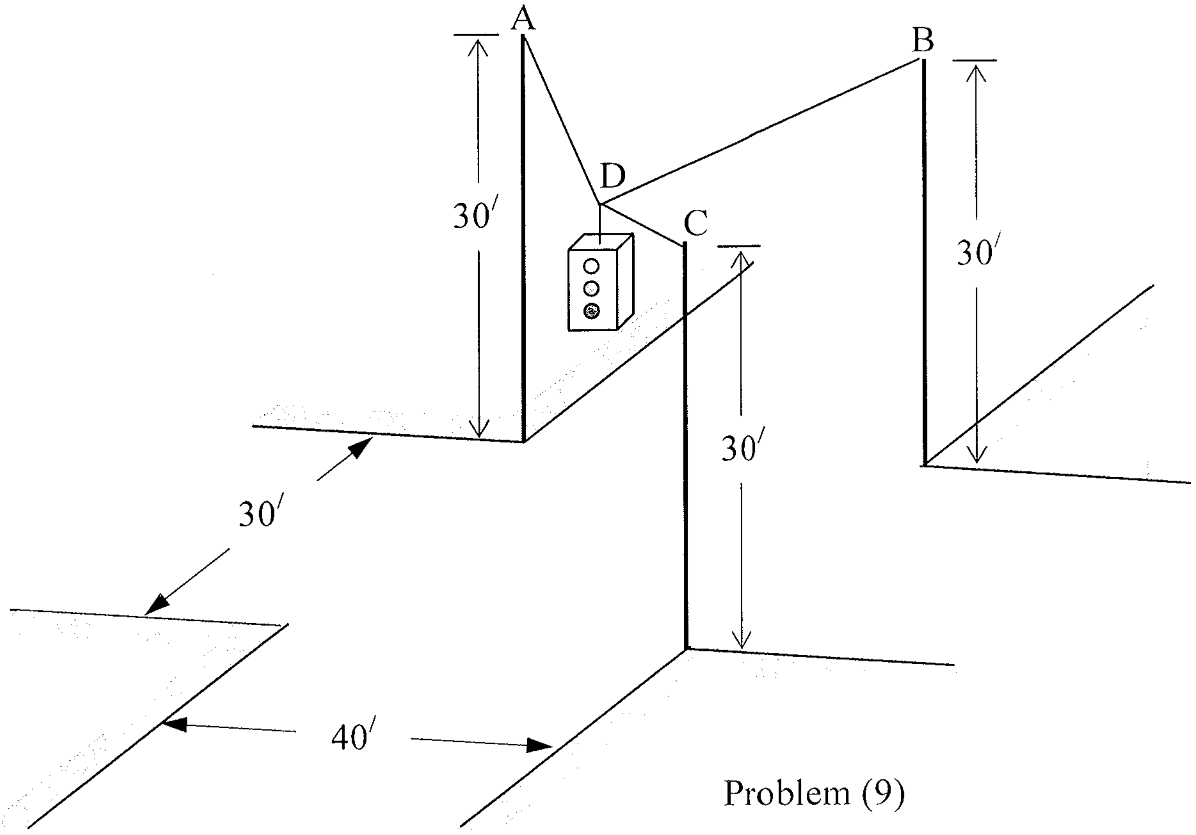
Answer (8): R_{AX} = 82.63518223 lb.; R_{AY} = 153.2088886 lb.; R_{AX} = 98.4807753 lb.; M_X = 612.8355545 ft-lb.; M_Y = 787.8462024 ft-lb. & M_Z = 247.9055467 ft-lb.;



Problem (8)

Problem (9): The traffic light weighs 100 lb. and is located at the center of the intersection such that point D is 25 ft above ground. A 30-lb wind load is acting to the left along the 30-ft-wide street. Determine the forces in the three cables supporting the traffic light.

Answer (9): T_{AD} = 235.8296525 lb. (tension); T_{BD} = 19.12132318 lb. (tension) & T_{CD} = 254.9509757 lb. (tension)



Problem (9)

Problem (10): A horizontal steel plate in the shape of an equilateral triangle, 12 ft. on a side and 6 inch thick is supported by three vertical cables, one attached at each vertex. The centre of gravity, which locates the line the action of the weight, lies at the intersection of the medians of the triangle. What is the tension in each cable? Steel weighs 490 lb. per cu. ft.

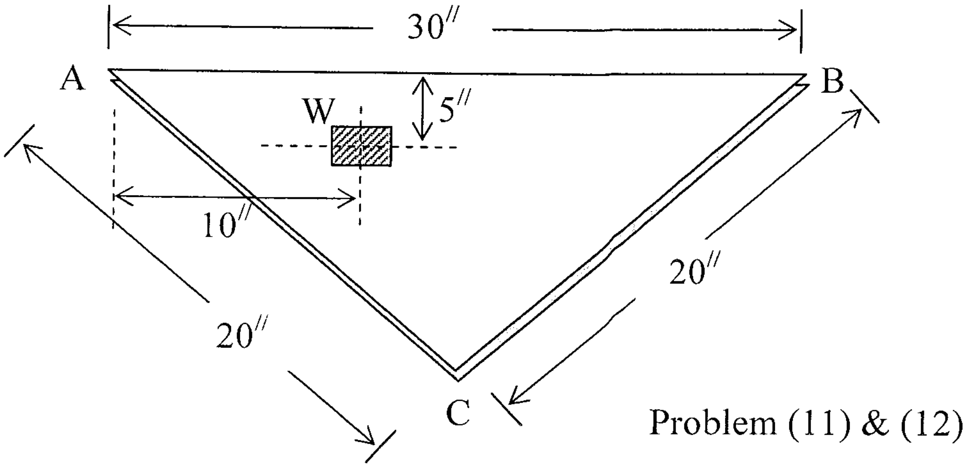
Answer (10): 5092.229374 lb. (tension)

Problem (11): A table whose top is triangular in shape has a load of W = 100 lb. as shown. What is the force on each leg at A, B & C?

Answer (11): R_A = 47.76844302 lb (↑); R_B = 14.43510968 lb (↑) & R_C = 37.7964473 lb (↑)

Problem (12): Same as problem (11) except that the weight of the triangular top is 30 lb. the centre of gravity, which locates the line of action of the weight, lies at the intersection of the medians of the triangle.

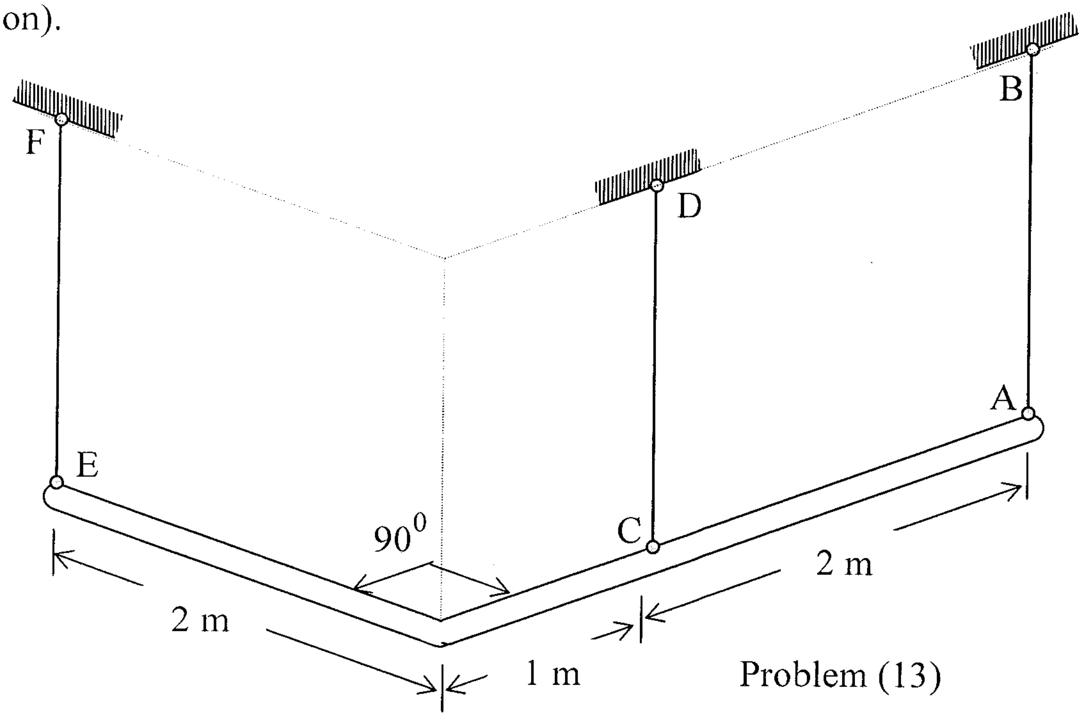
Answer (12): R_A = 57.76844302 lb (↑); R_B = 24.43510968 lb (↑) & R_C = 47.7964473 lb (↑)



Problem (11) & (12)

Problem (13): A right-angle bend pipe is supported in a horizontal plane with the help of three vertical wires AB, CD and EF. The pipe weighs 40 kg/meter. Calculate the values of tension in the wires AB, CD and EF.

Answer (13): F_{AB} = 10 kg. (tension); F_{CD} = 150 kg. (tension). & F_{EF} = 40 kg. (tension).



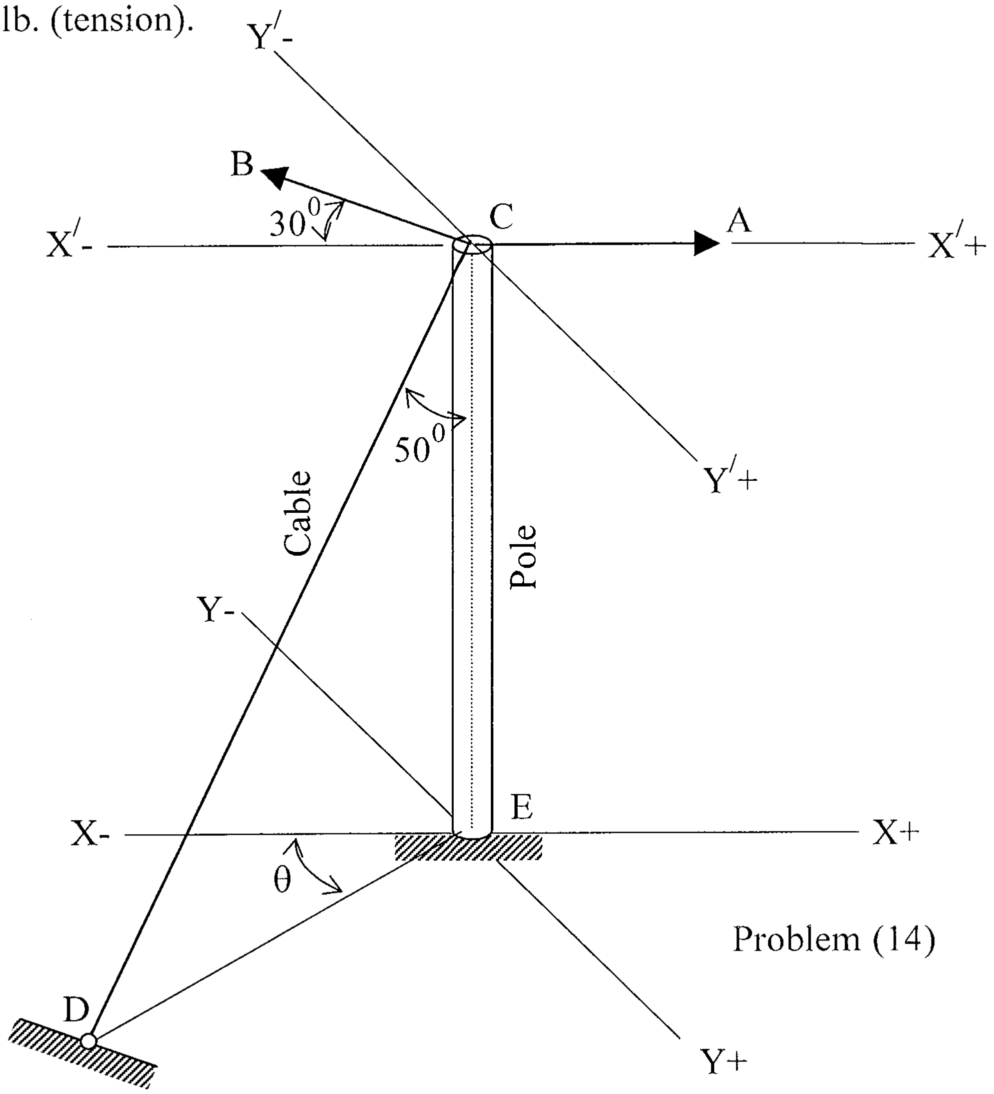
Problem (13)

Problem (14): Two cables AC and BC terminate on a pole and exert forces in a horizontal plane at $X^{/}Y^{/}$ at C. The tension in the cables AC and BC are 6000 lb. and 4000 lb. respectively. The guy cable makes an angle of 500 with the pole. Fine the value of the angle θ, tension in the cable CD and force in the pole CE.

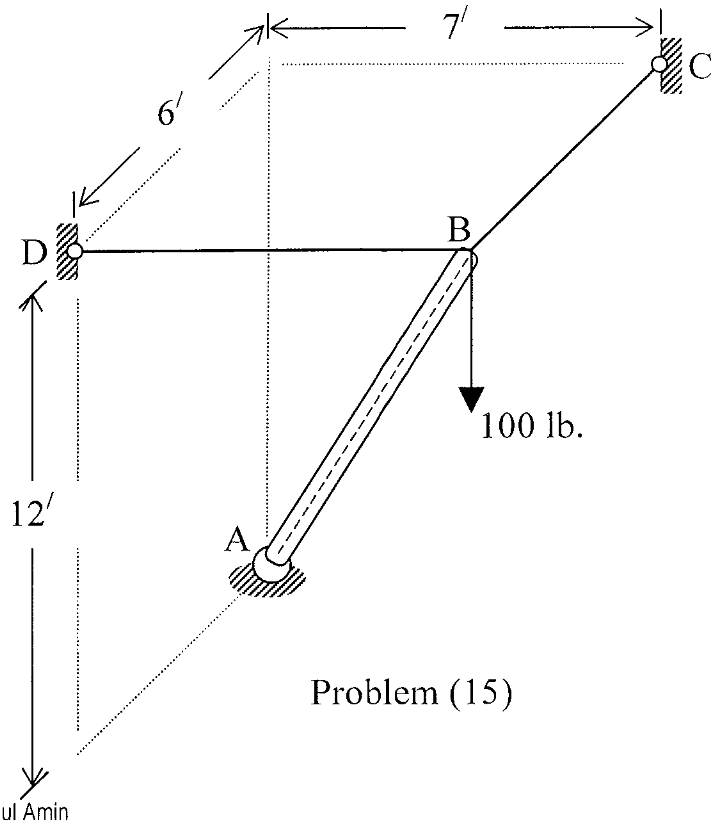
Answer (14): $\theta = 38.26196662^0$; F_{CD} = 4216.037248 lb. (tension) & F_{CE} = 2710.016505 lb. (compression)

Problem (15): A mast AB, supported by a spherical socket at A and horizontal guy wires BC & BD, carries a vertical load of 100 lb. at B. the wires BC & BD intersect at B at right angle in a horizontal plane. Find the force in the mast AB and tension in the wires BC & BD. Neglect weight of the mast AB.

Solution (15): F_{AB} = 126.1062163 lb. (compression); F_{BC} = 50 lb. (tension). & F_{BD} = 58.33333333 lb. (tension).



Problem (14)



Problem (15)

Chapter Eleven Mass Moment of Inertia

Definition: If a body rotates about an axis as shown in figure 1, the equations of motion involve the expression $\int r^2 dm$, where dm is any differential element of the mass and where r is the perpendicular distance from dm to the axis of rotation.

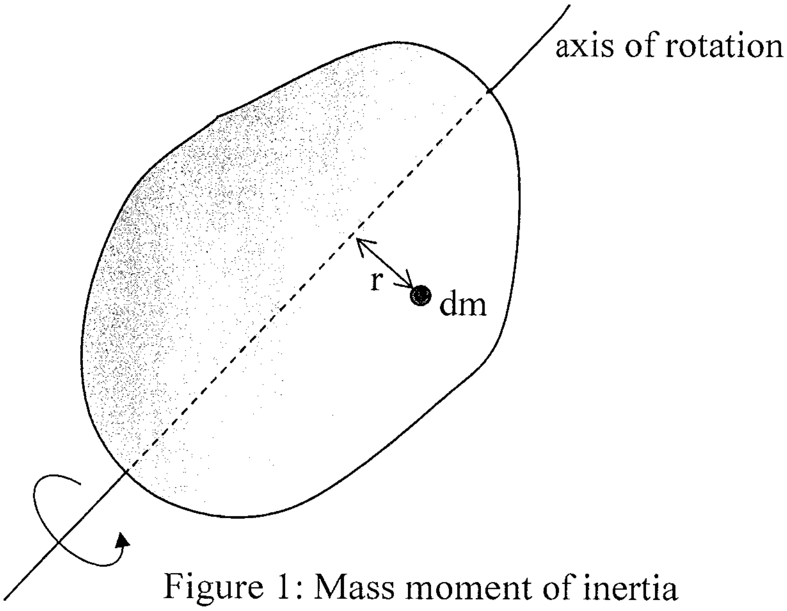


Figure 1: Mass moment of inertia

$$I_m = \int r^2 dm$$

The integral which is evaluated for the entire mass m is called the moment of inertia of the mass with respect to the axis of rotation.

Unit of Mass Moment of Inertia: Since, mass moment of inertia is product of mass and square of distance, its unit can be derived as given below.

In M.K.S system

$$\text{mass} \times \text{distance}^2 = \frac{\text{weight}}{\text{gravitational acceleration}} \times \text{distance}^2$$

$$= \frac{\text{Newton}}{\text{meter/second}^2} \times \text{meter}^2$$

$$= \text{Newton} - \text{meter} - \text{second}^2 = \text{N} - \text{m} - \text{sec}^2$$

In C.G.S system

$$\text{mass} \times \text{distance}^2 = \frac{\text{weight}}{\text{gravitational acceleration}} \times \text{distance}^2$$

$$= \frac{\text{Dyne}}{\text{centimeter/second}^2} \times \text{centimeter}^2$$

$$= \text{Dyne} - \text{centimeter} - \text{second}^2 = \text{Dyne} - \text{cm} - \text{sec}^2$$

In F.P.S system

$$\text{mass} \times \text{distance}^2 = \frac{\text{weight}}{\text{gravitational acceleration}} \times \text{distance}^2$$

$$= \frac{\text{Poundal}}{\text{feet/second}^2} \times \text{feet}^2$$

$$= \text{Poundal} - \text{foot} - \text{second}^2 = \text{Poundal} - \text{ft} - \text{sec}^2$$

However, engineers are accustomed to using the pound as a unit of force. As a result, the mass m, the units of which are then derived, is found to be W/g, where W is the spring-balance weigh of the body (the force of gravity) and g is the acceleration of gravity at the point on the earth's surface where the weight is terminated. For most engineering purposes, it is sufficiently accurate to consider g constant, with a value of g = 32.2 feet/second^2 (fps^2). Thus, with the weight expressed in pounds, the corresponding derived units of mass are obtained as –

$$m = \frac{W}{g}$$

$$\text{mass} = \frac{\text{weight}}{\text{gravitational acceleration}}$$

$$= \frac{\text{Pound}}{\text{feet/second}^2} = \frac{\text{Pound} - \text{second}^2}{\text{feet}} = \frac{\text{lb} - \text{sec}^2}{\text{ft}} = \text{slug}$$

A unit that is called a slug (a slug = lb.-sec.^{2} per ft.). Now since the linear unit is involved in our conception of mass when we use g in feet per second-second (fps^2), the moment arm r must also be expressed in feet in order to keep the units consistent. Consequently, the units of moment of inertia ($\int r^2 dm$) are

$$= \frac{\text{Pound}}{\text{feet/second}^2} \times \text{feet}^2 = \frac{\text{lb} - \text{sec}^2}{\text{ft}} \times \text{ft}^2 = \text{slug - ft}^2$$

Recall that we frequently use the specific weight w of material in pound per cubic foot and the mass density ρ is then w/g slugs per cubic foot.

[*Slug (gee-pound) – the mass of a body whose weight is 32.2 lb. at the sea level is called a slug or gee-pound]

Radius of Gyration (k): Radius of gyration is that distance which when squared and multiplied with total mass of the body gives the mass moment of inertia of the body. Thus, if I is moment of inertia of a body of mass M about an axis, then its radius of gyration k about that axis is given by the relation:

$$k = \sqrt{\frac{I}{m}} \text{ or } I = mk^2 = \int r^2 dm$$

A physical meaning may be assigned to the term. Radius of gyration is the distance at which the entire mass can be assumed to be concentrated such that the moment of inertia of the actual body and the concentrated mass is the same.

Moment of Inertia about Planes & Axes: The moment of inertia of a mass m may be with respect to either a plane or axis. In figure 2, let a differential element of mass dm have the dimensions dx, dy, and dz, with the dz dimension parallel to z axis. Since all parts of this element are then at the same distance y from the xz plane, and at the same distance x from the yz plane, the moments of inertia with respect to these planes are

$$I_{xz} = \int y^2 dm \text{ and } I_{yz} = \int x^2 dm$$

(Observe that while all parts of this differential element are the same distance from the z axis, all parts are not at the same distance from the x and y axes). Moreover, with all parts of the element dm at the same distance r from the z axis, the moment of inertia with respect to that axis is

$$I_z = \int r^2 dm$$

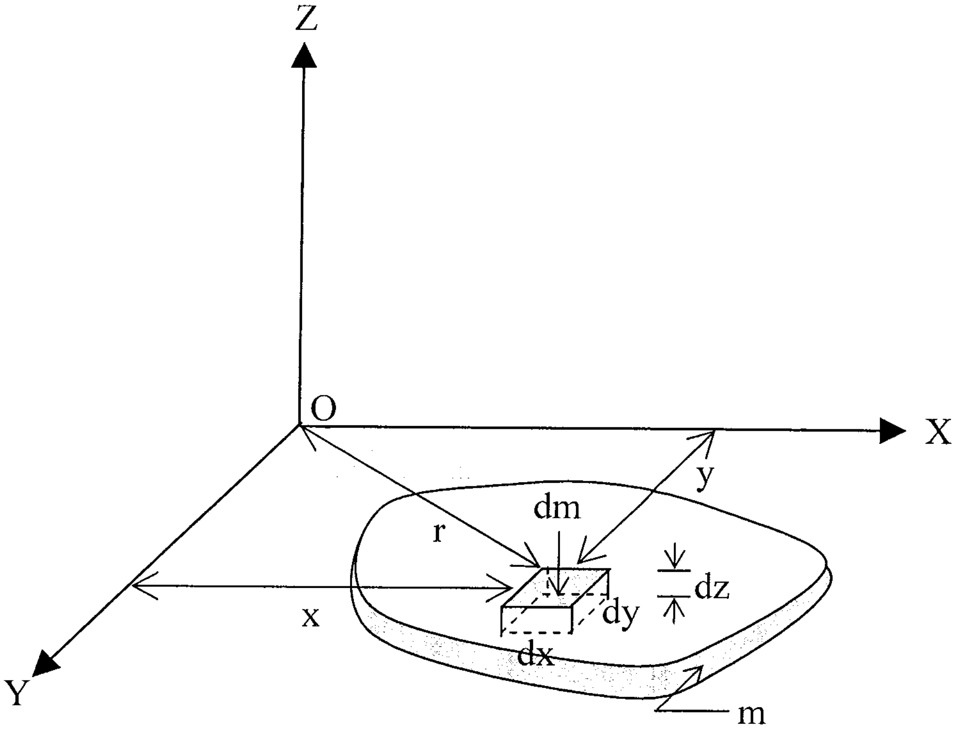


Figure 2: Rectangular and polar mass moment of inertia

Since, from the geometry of the figure, $r^2 = x^2 + y^2$, we see that

$$I_z = \int r^2 dm = \int (x^2 + y^2) dm = \int (y^2 + x^2) dm = I_{xz} + I_{yz}$$

that is, the sum of the inertia of a mass with respect to two planes at right angles to each other is equal to the moment of inertia of the mass with respect to the axis formed by the intersection of the planes. This rule is sometimes helpful in determining a moment of inertia about an axis.

Transfer formula - Parallel Axis Theorem: the moment of inertia of a body about an axis at a distance d and parallel to a centroidal axis is equal to sum of moment of inertia about centroidal axis and product of mass and square of distance of parallel axis.

$$(I)_{\text{about any axis}} = (I)_{\text{about a parallel axis at centroid}} + md^2$$

=>> $I = \bar{I} + md^2$

Where, I = any axis about which mass moment of inertia is needed (i.e., mass moment of inertia with respect to any non-centroidal axis)

$\bar{I}$ = mass moment of inertia about centoidal axis (parallel to the axis about which moment of inertia is needed)

m = mass

d = perpendicular distance between centroidal axis and reference axis.

Now, $I = \bar{I} + md^2$

$mk^2 = m\bar{k}^2 + md^2$

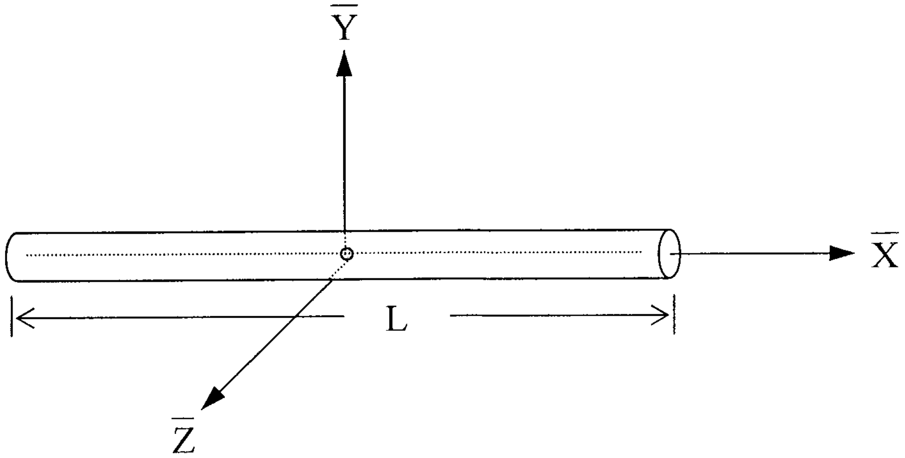
$k = \bar{k} + d^2$

The mass moments of inertia given below are the moments of inertia with respect to the centriodal axes. To determine the moments or products of inertia with respect to any other set of axes, the parallel axis theorem or rotation axis theorem may be employed.

The slender rod

$\overline{I_{xx}} = 0$

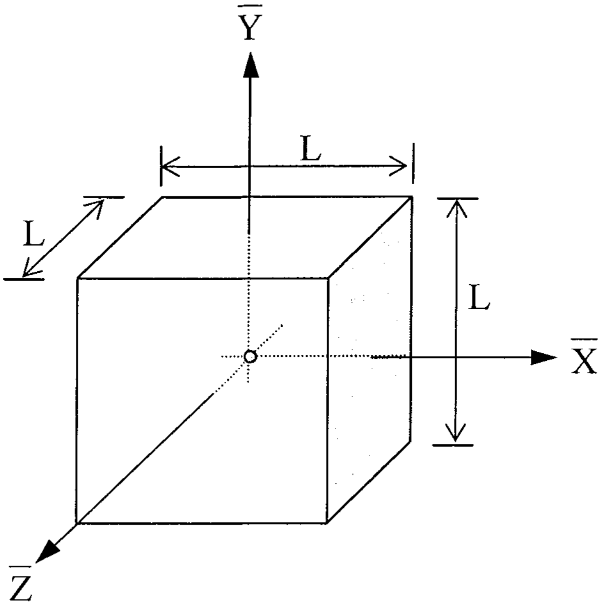
$\overline{I_{yy}} = \overline{I_{zz}} = \frac{mL^2}{12}$



The cube

$m = \rho \times L^3$

$\overline{I_{xx}} = \overline{I_{yy}} = \overline{I_{zz}} = \frac{mL^2}{6}$



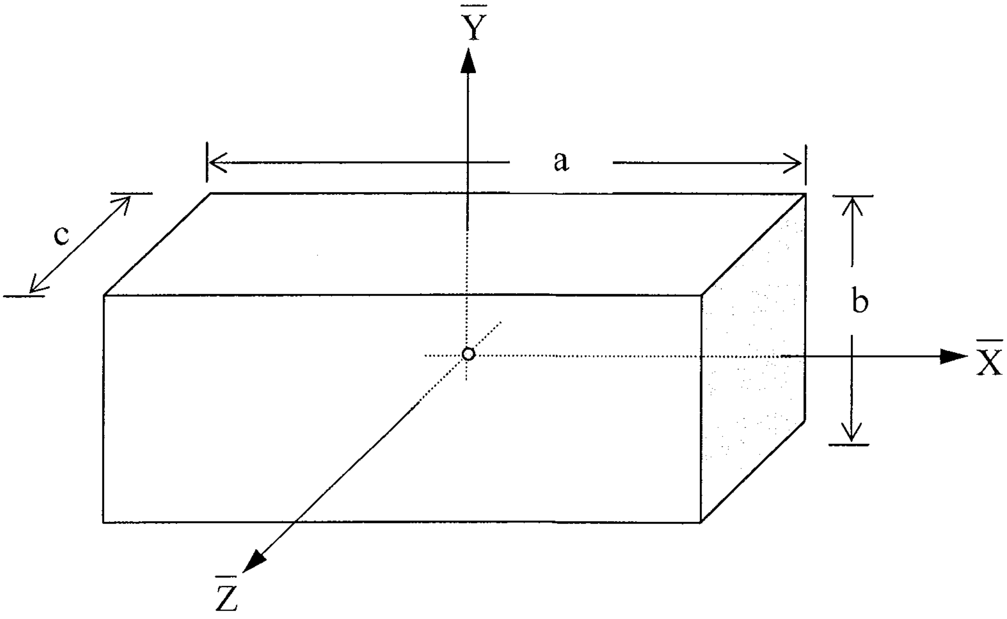
The rectangular prism (rectangular parallelepiped)

$m = \rho \times abc$

$\overline{I_{xx}} = \frac{m(b^2 + c^2)}{12}$

$\overline{I_{yy}} = \frac{m(a^2 + c^2)}{12}$

$\overline{I_{zz}} = \frac{m(a^2 + b^2)}{12}$



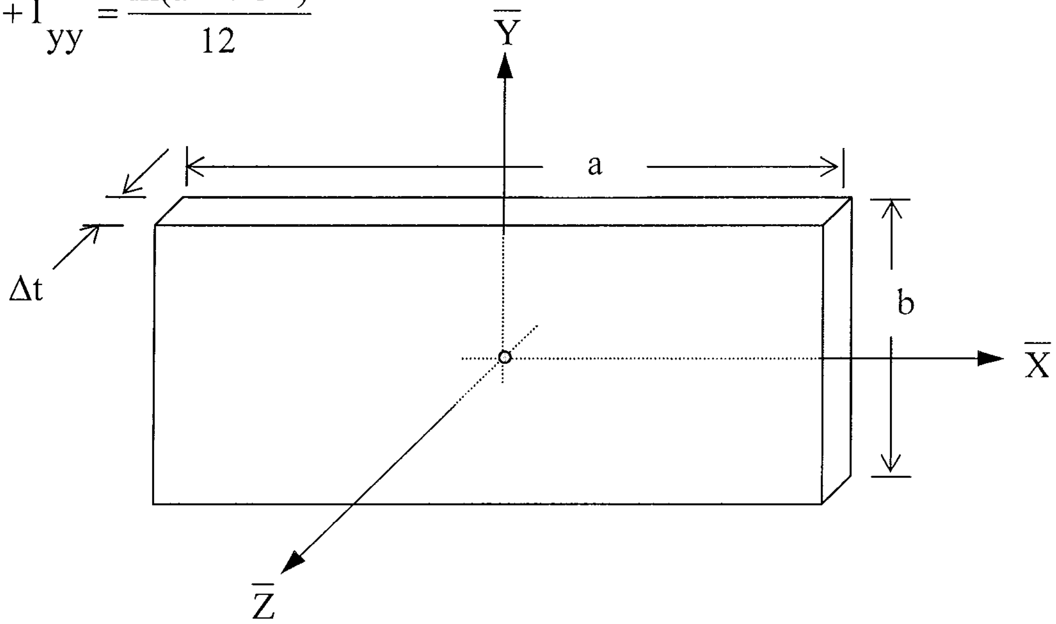
The thin rectangular plate

$$m = \rho \times (\Delta t)ab$$

$$\overline{I_{xx}} = \frac{mb^2}{12}$$

$$\overline{I_{yy}} = \frac{ma^2}{12}$$

$$\overline{I_{zz}} = \overline{I_{xx}} + \overline{I_{yy}} = \frac{m(a^2 + b^2)}{12}$$

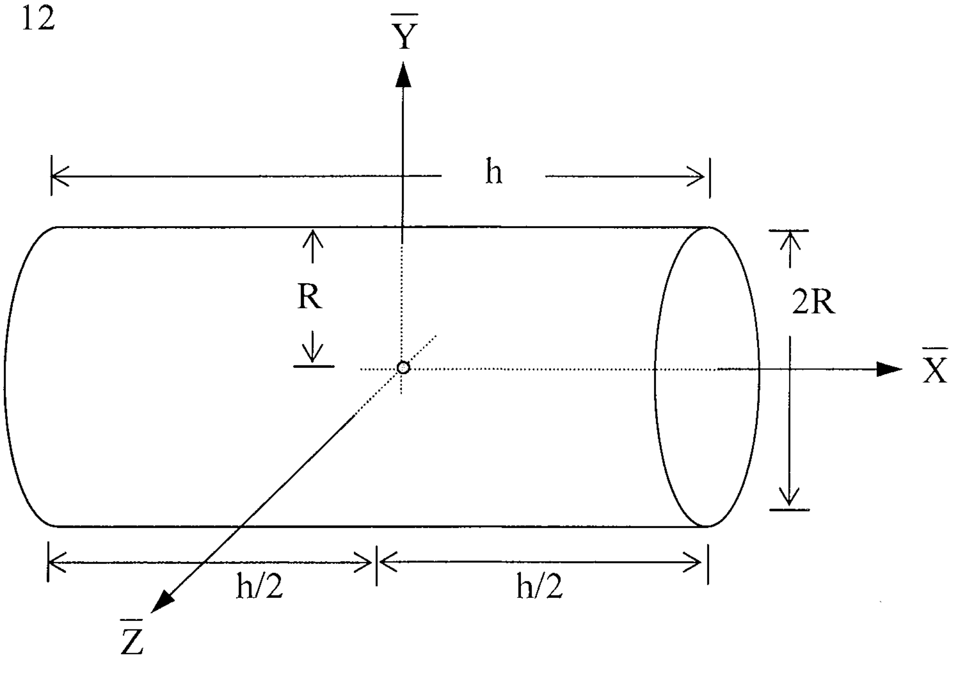


The cylinder

$$m = \rho \times \pi R^2 h$$

$$\overline{I_{xx}} = \frac{mR^2}{2}$$

$$\overline{I_{yy}} = \overline{I_{zz}} = \frac{m(3R^2 + h^2)}{12}$$

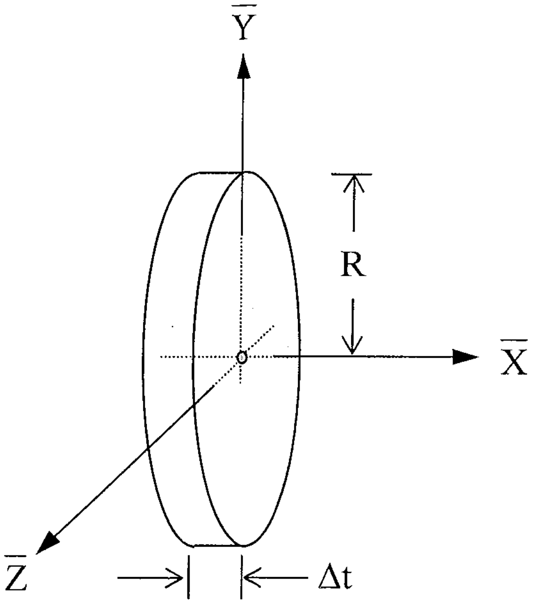


The thin disk

$$m = \rho \times \pi R^2 (\Delta t)$$

$$\overline{I_{xx}} = \frac{mR^2}{2}$$

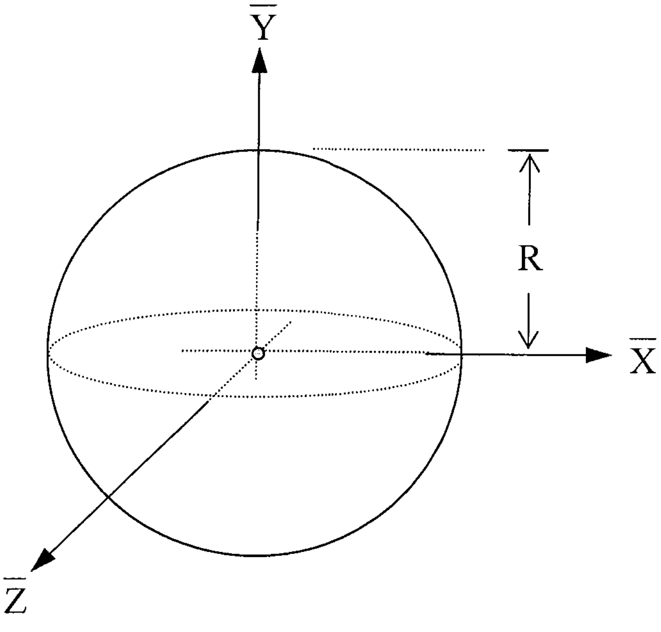
$$\overline{I_{yy}} = \overline{I_{zz}} = \frac{mR^2}{4}$$



The sphere

$$m = \rho \times \frac{4\pi R^3}{3}$$

$$\overline{I_{xx}} = \overline{I_{yy}} = \overline{I_{zz}} = \frac{2mR^2}{5}$$

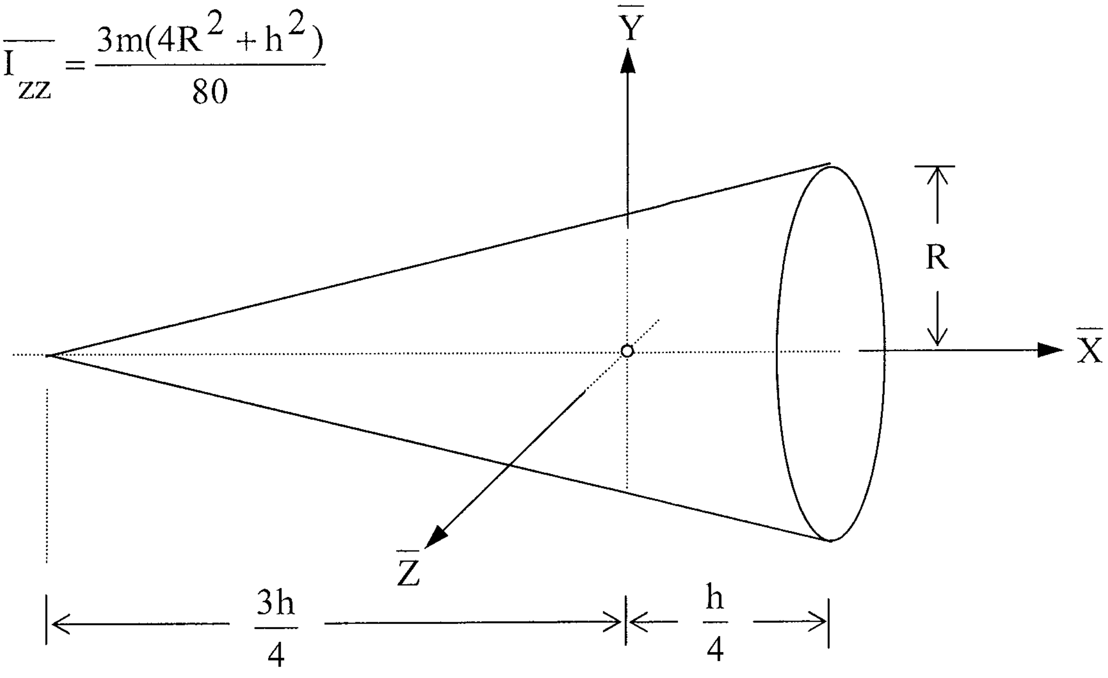


The cone

$$m = \rho \times \frac{1}{3}\pi R^2 h$$

$$\overline{I_{xx}} = \frac{3mR^2}{10}$$

$$\overline{I_{yy}} = \overline{I_{zz}} = \frac{3m(4R^2 + h^2)}{80}$$



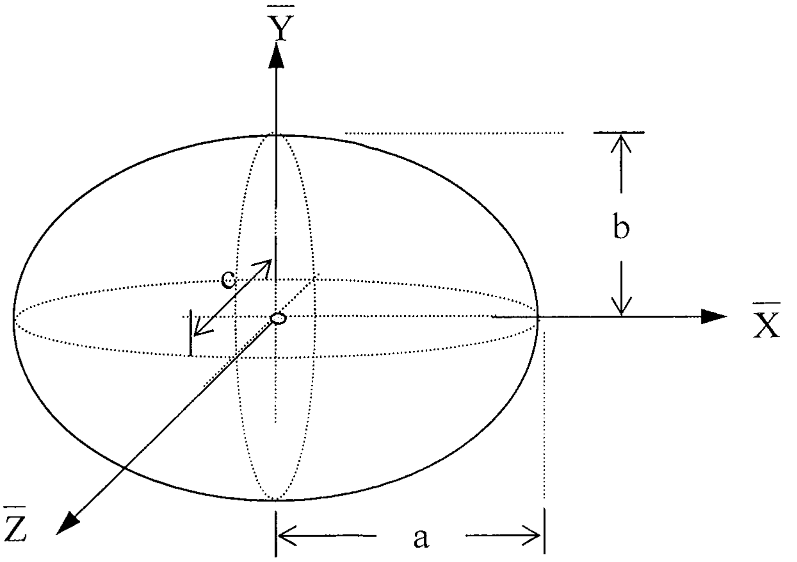
The ellipsoid

$$m = \rho \times \frac{4\pi abc}{3}$$

$$\overline{I_{xx}} = \frac{m(b^2 + c^2)}{5}$$

$$\overline{I_{yy}} = \frac{m(c^2 + a^2)}{5}$$

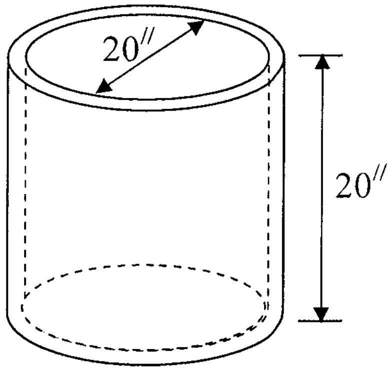
$$\overline{I_{zz}} = \frac{m(a^2 + b^2)}{5}$$



Problems

Problem (1): An idealized version of the bucket of a mixer is shown. Determine the mass moment of inertia about the vertical axis when the bucket is empty, and when one-half full of aggregate weighing 150 lb/cft. The bucket material weighs 0.284 lb/cubic inch and the bucket has a wall and bottom thickness of 0.25 inch.

Answer (1): 2.238090781 slug-ft^2 & 5.142020748 slug-ft^2



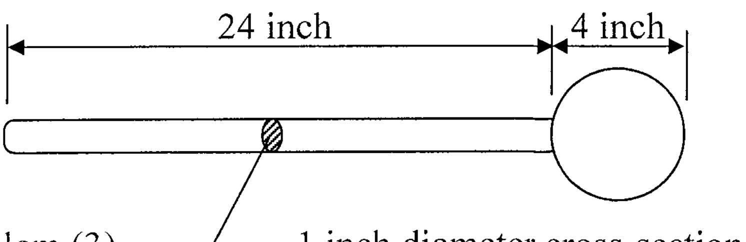
Problem (1)

Problem (2): A brass cylinder 4 inch in diameter and 8 inch long is rotating about geometric axis. What is the value of its moment of inertia? It weighs 570 lb/cft.

Answer (2): 0.014303509 slug-ft^2

Problem (3): A steel ball (w = 490 lb. per cft.) 4 inch in diameter is mounted at the end of a slender steel bar 1 inch in diameter and 2 feet long, as shown. What is the moment of inertia of the system about the vertical axis through the left end of the bar?

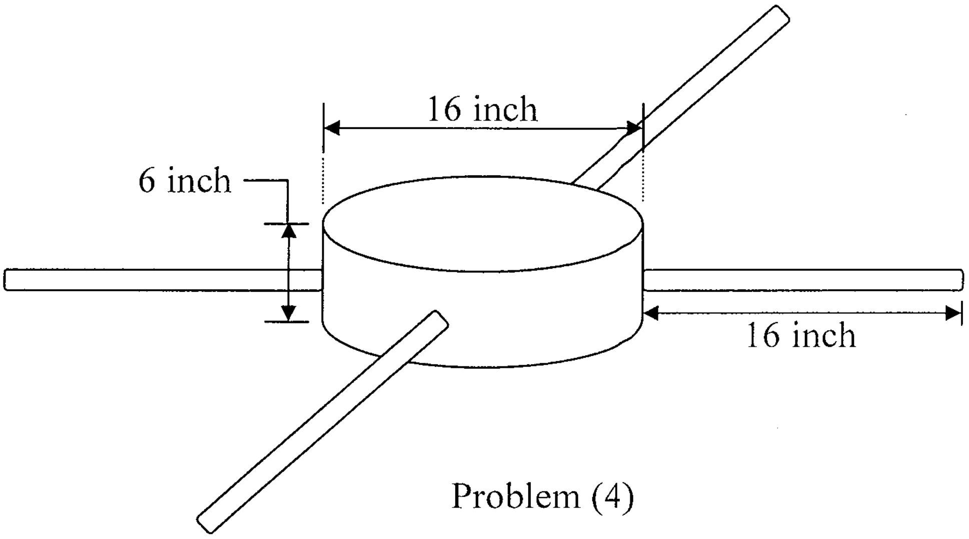
Answer (3): 1.609956156 slug-ft^2



Problem (3)

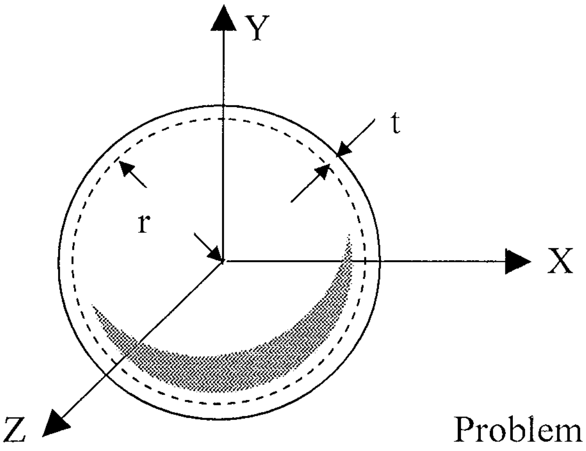
Problem (4): The solid central hub shown has four similar arms, each weighing 0.83 lb, attached to it as shown. Assuming the hub has a weight density of 490 lb/cft, find the mass moment of inertia about the vertical centroidal axis.

Answer (4): 2.788529101 slug-ft^2



Problem (4)

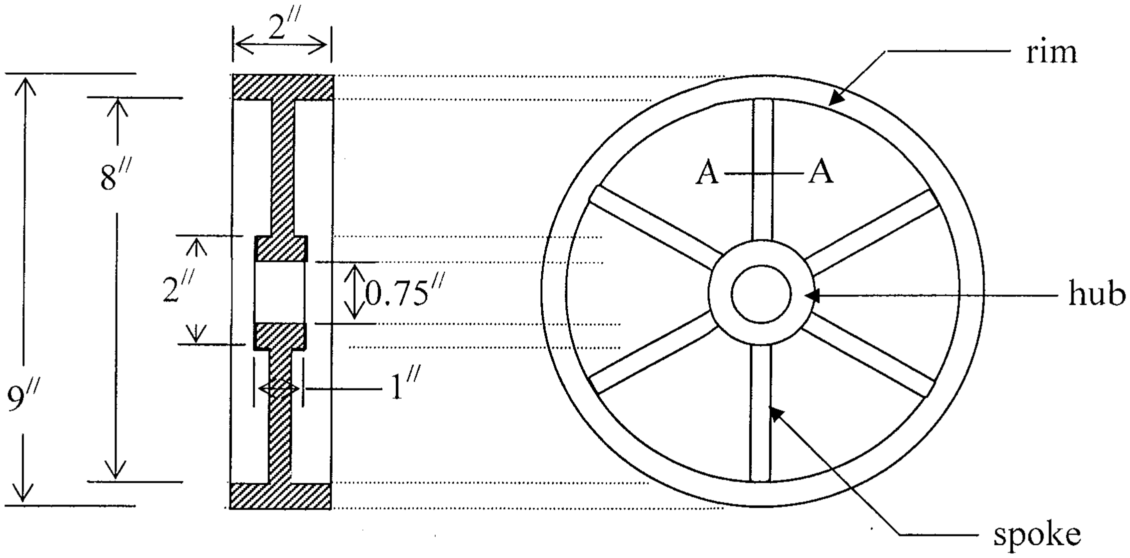
Problem (5): A thin-shelled sphere shown has thickness t and inside radius r. show that the mass moment of inertia about a diameter is $\frac{2}{3}mr^2$.



Problem (5)

Problem (6): Determine the mass moment of inertia of the fly-wheel shown. The material weighs 450 lb/cft. The spokes are 0.5 inch in diameter and may be treated as slender bars.

Answer (6): 0.028658941 slug-ft^2



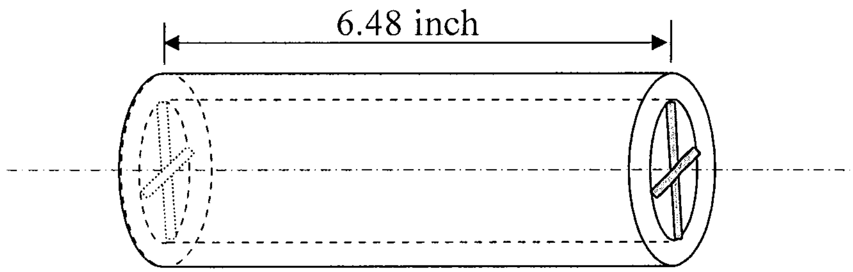
Problem (6)

▨ 0.50'' diameter, 6 spokes

Section A-A

Problem (7): The tube shown rotates about its geometric axis. Neglecting the weights of the end supports, determine the mass moment of inertia of the tube which weighs 0.31 lb/cubic inch.

Answer (7): 0.007159666 slug-ft^2



Problem (7)

inner radius = 1.08'' & outer radius = 2.10''

Problem (8): A slender bar 8 inch long and 0.20 square inch in area is rotation about an axis through one end. What is its mass moment of inertia, assuming a uniform density of 0.28 lb/cubic inch?

Answer (8): 0.002061191 slug-ft^2

Problem (9): An 8-inch monel metal (w = 555 lb. per cft.) disk with a 3-inch co-axial hole is used as a flywheel. If the disk is 1 inch thick, what is its radius of gyration?

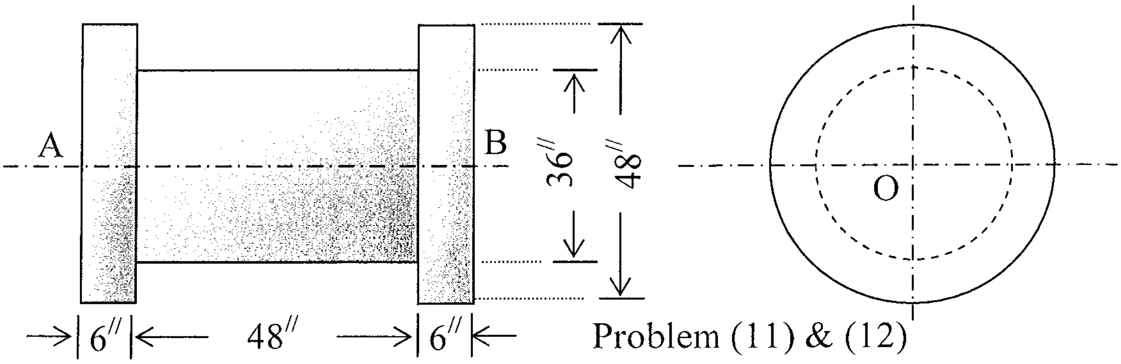
Answer (9): 3.020761493 inch.

Problem (10): Determine the moment of inertia of the steel spool about the geometric axis AB. Steel weighs 490 lb. per cft.

Answer (10): 866.4990607 slug-ft^2

Problem (11): The same as problem (10) except that there is a cylindrical hole 2 ft. in diameter whose axis coincides with AB. The hole passes entirely through the spool.

Answer (11): 627.4648371 slug-ft^2



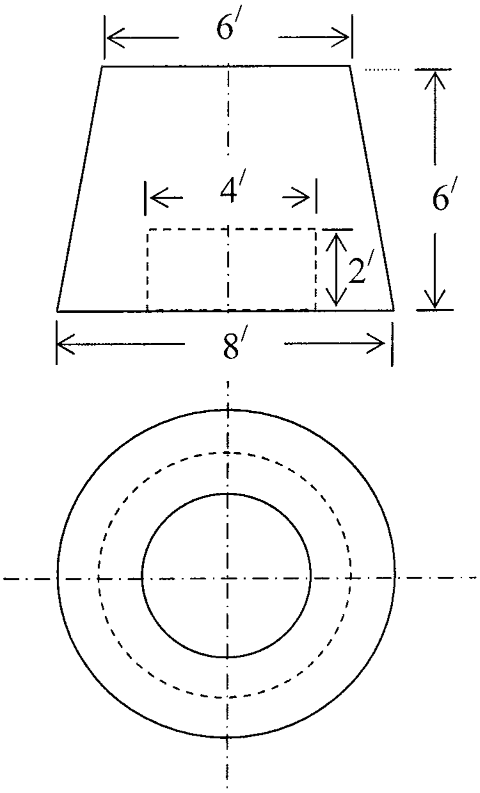
Problem (11) & (12)

Problem (12): What is the moment of inertia about a diameter of a hollow cast-iron sphere with an outside radius of 4 ft and inside radius of 2 ft? cast iron weighs 450 lb. per cubic feet.

Answer (12): 23,228.27264 slug-ft^2

Problem (13): Find the moment of inertia about the geometric axis of the cast-iron (490 lb. per cft) frustum of a cone which has a short 4 x 2 ft. cylindrical hole in the base.

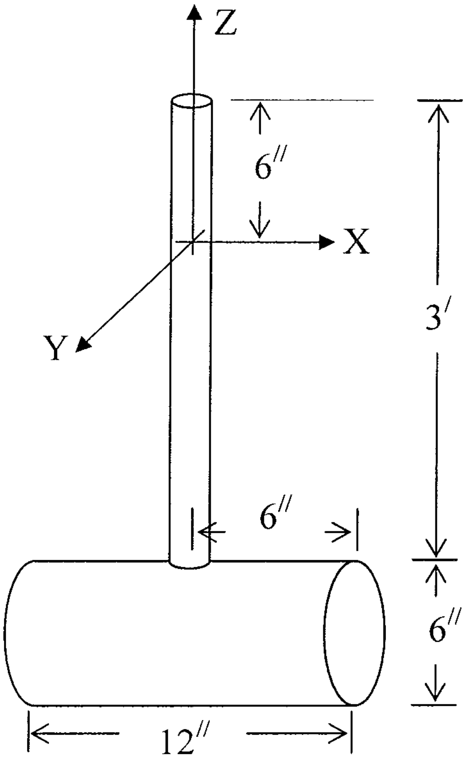
Answer (13): 21673.37792 slug-ft^2



Problem (13)

Problem (14): The wood handle, of the mallet of is 3 ft. long weighs 3.14 lb., and has a uniform cross-section. The head, weighing 16.1 lb., is a wood cylinder of diameter is 6 inch. Find the radius of gyration of the body with respect to the y-axis. The mallet head with the handle is homogeneous.

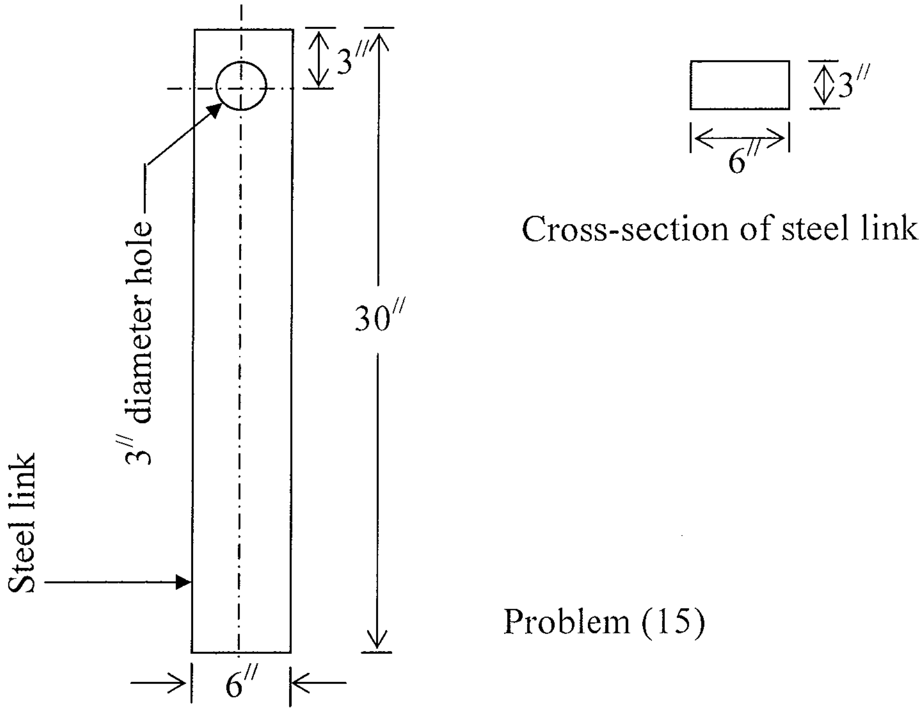
Answer (14): 2.614573342 ft.



Problem (14)

Problem (15): A steel link (490 lb. per cft) rotates about the axis of the 3-inch hole. Allowing for the hole, compute the moment of inertia and radius of gyration of the link about the axis of rotation (Do not make the approximation of a slender rod). Compare this answer with that obtained by using the approximation for a slender rod and neglecting the effect of the hole.

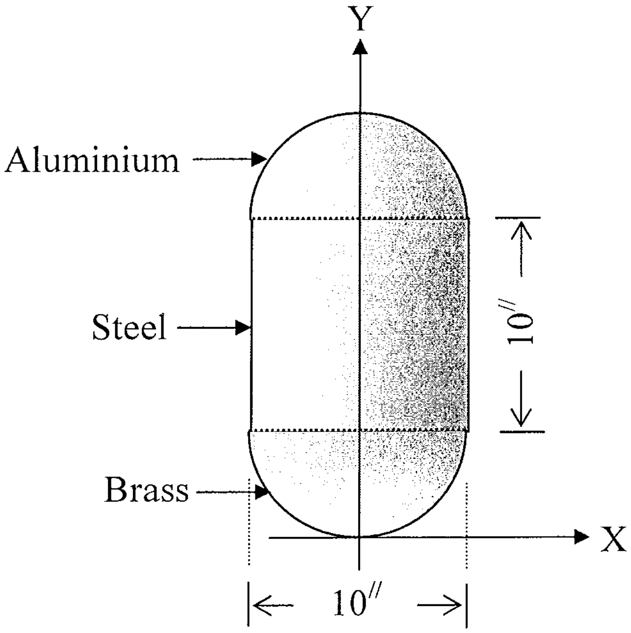
Answer (15): 7.138584322 slug-ft^2, 1.25863883 ft.; 7.23223732 slug-ft^2, 1.2332207 ft



Problem (15)

Problem (16): A short steel cylinder (490 lb. per cft) is 10 inch in diameter & 10 inch long. One end attached to an aluminium hemisphere (170 lb. per cft) & the other end is attached to a brass hemisphere (530 lb. per cft). (a) Locate the mass center of this composite body. (b) Find the moment of inertia of this body about a gravity axis parallel to the x axis shown.

Answer (16): (a) $\bar{y} = 8.859447004$ inch (from bottom); (b) 1.85 slug-ft^2



Problem (16)

Direct Determination of Mass Moment of Inertia: See page no. 72-74 of book Mechanical Properties of Matter – S. G. Starling, McGraw-Hill Book Company.